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Restricted methods in symmetrical linear regression models

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Abstract

In this paper we discuss the problem of testing equality and inequality constraints in symmetrical linear regression models. This class of models includes all symmetric continuous distributions, such as normal, Student-*t*, Pearson VII, power exponential and logistic, among others. It is commonly used for the analysis of data containing influential or outlying observations with responses supposedly normal. Iterative processes for evaluating the parameters under equality and inequality constraints are presented. The asymptotic null distribution of three asymptotically equivalent one-sided tests is showed to be invariant with the symmetrical error. A sensitivity study to investigate the robustness of the maximum likelihood estimates from some symmetrical models against high leverage and influential observations is presented. An illustrative example with presence of influential observations on the decisions from the statistical tests of different symmetrical models is given. The robustness aspects of such models are also discussed.

Keywords: Hypothesis testing; Symmetric distributions; One-sided tests; Order restrictions; Restricted estimation; Robustness

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1. Introduction

In this paper we discuss two situations of testing restricted hypotheses in symmetrical linear regression models. First, the problem of testing linear equality hypothesis $H_0 : \mathbf{C}\boldsymbol{\beta}=\mathbf{d}$ against the linear inequality hypothesis $H_2 : \mathbf{C}\boldsymbol{\beta} \geqslant \mathbf{d}$, with at least one strict inequality in H_2 (case 1) is treated, and then, $H_2 : \mathbf{C}\boldsymbol{\beta} \geqslant \mathbf{d}$ against $\mathbb{R}^p$ - H_2 (case 2) is discussed. The problem of testing one-sided alternatives was originally treated by Bartholomew (1959a, b) for independent normal models and extended by Kudo (1963) for multivariate normal models. Nuesch (1966) also investigates this problem in normal models while Perlman (1969) extends the results for a more general class of multivariate normal models. Gourieroux et al. (1982) discuss the asymptotic null distribution of three asymptotically equivalent one-sided tests in multivariate normal models when the variance-covariance matrix may depend on a finite number of unknown parameters. Wolak (1987) proposes exact one-sided tests for multivariate normal models and Wolak (1989) extends the results from Gourieroux et al. (1982) for more general restricted hypotheses. Moving away from the normal case, Kodde and Palm (1986) and Silvapulle and Silvapulle (1995), for instance, present Wald and score type tests that may be applied for testing equality and inequality restrictions in general multivariate regression models. In case 2, the main difficulty is when the information matrix depends on the parameter $\boldsymbol{\beta}$. A consequence of this fact is that we should search through the set of null parameters for least favorable points. Wolak (1991) proposes a lemma in which a methodology to find a least favorable region is presented. An excellent review on this subject may be found in the book by Robertson et al. (1988) (see also, Sen and Silvapulle, 2002).

The paper is organized as follows. In Section 2 we discuss the unrestricted parameter estimation in symmetrical linear models. Iterative processes for evaluating the maximum likelihood restricted estimates under equality and inequality constraints are given in Section 3. Section 4 contains the expressions as well as the asymptotic null distribution of three asymptotically equivalent one-sided tests. In Section 5 a sensitivity study to investigate the robustness of the maximum likelihood estimates from some symmetrical models against high leverage and influential observations is given. An illustrative example in which influential observations change the decisions from the statistical tests of different symmetrical models is presented in Section 6. The robustness aspects of such models are discussed. The last section deals with some concluding remarks.

2. Symmetrical linear models

Suppose $Y_1, \ldots, Y_n$ independent random variables with density function of Y_i given by

$$f_{y_i}(y) = \frac{1}{\sqrt{\phi}} g\{(y-\mu_i)^2/\phi\}, \tag{1}$$

$y \in \mathbb{R}$, where the function $g : \mathbb{R} \to [0, \infty)$ is such that $\int_0^\infty g(u)\,\mathrm{d}u < \infty$. The function $g(\cdot)$ is typically known as the density generator. We will denote $Y_i \sim \mathrm{S}(\mu_i, \phi)$. The symmetrical