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A general asset–liability management model for the efficient simulation of portfolios of life insurance policies

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Abstract

New regulations and a stronger competition have increased the importance of stochastic asset–liability management (ALM) models for insurance companies in recent years. In this paper, we propose a discrete time ALM model for the simulation of simplified balance sheets of life insurance products. The model incorporates the most important life insurance product characteristics, the surrender of contracts, a reserve-dependent bonus declaration, a dynamic asset allocation and a two-factor stochastic capital market. All terms arising in the model can be calculated recursively which allows an easy implementation and efficient simulation. Furthermore, the model is designed to have a modular organization which permits straightforward modifications and extensions to handle specific requirements. In a sensitivity analysis for sample portfolios and parameters, we investigate the impact of the most important product and management parameters on the risk exposure of the insurance company and show that the model captures the main behaviour patterns of the balance sheet development of life insurance products.

JEL classification: C15; G22; G32

MSC: IB10; IM22

Keywords: Asset–liability management; Participating policies; Numerical simulation

1. Introduction

The scope of asset–liability management (ALM) is the responsible administration of the assets and liabilities of insurance contracts. Here, the insurance company has to attain two goals simultaneously. On the one hand, the available capital has to be invested profitably, usually in bonds but also, up to a certain percentage, in stocks (asset management). On the other hand, the obligations against policyholders, which depend on the specific insurance policies, have to be met (liability management). In this paper, we focus on portfolios of participating (with-profit) policies which make up a significant part of the life insurance market. The holder of such a policy gets a fixed guaranteed interest and, in addition, a variable reversionary bonus which is annually added to the policyholder's account and allows the policyholder to participate in the investment returns of the company. Thereby, the insurance company has to declare in each year which part of the investment returns is given to the policyholders as reversionary bonus, which part is saved in a reserve account for future bonus payments and which part is kept by the shareholders of the company. These management decisions depend on the financial situation of the company as well as on strategic considerations and legal requirements. A maximization of the shareholders' benefits has to be balanced with a competitive bonus declaration for the policyholders. Moreover, the exposure of the company to financial, mortality and surrender risks has to be taken into account. These problems, which easily become quite complex due to the wide range of guarantees and option-like features of insurance products and management rules, are investigated with the help of ALM analyses. In this context, it is necessary to estimate the medium- and long-term development of all assets and liabilities

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as well as the interactions between them and to determine their sensitivity to the different types of risks. This can either be achieved by the computation of particular scenarios (stress tests) which are based on historical data, subjective expectations, and guidelines of regulatory authorities or by a stochastic modelling and simulation of the market development, the policyholder behaviour and all accounts involved.

In recent years, the latter approach has attracted more and more attention as it takes financial uncertainties more realistically into account than an analysis of a small number of deterministically given scenarios. Additional importance arises from the current need of insurance companies to move from an accounting based on book values to a market-based, fair value accountancy standard as required by Solvency II and the International Financial Reporting Standard (IFRS); see, e.g., Jorgensen (2004). This task can be achieved by performing stochastic simulations of ALM models in a risk neutral environment. Much effort has been spent on the development of such models in the last few years; see, e.g., Albizzati and Geman (1994), Bacinello (2001, 2003), Briys and Varenne (1997), De Felice and Moriconi (2005), Goecke (2003), Grosen and Jorgensen (2000, 2002), Kling et al. (2007), Miltersen and Persson (2003), Moller and Steffensen (2007), Tanskanen and Lukkarinen (2003) and the references therein. Here, most authors focus on the fair valuation and contract design of unit-linked and participating life insurance policies. Exceptions are Goecke (2003) and Kling et al. (2007) where the financial risks and returns of participating policies are analysed under the real world probability measure. Most of the articles in the existing literature (exceptions are Bacinello (2001, 2003), De Felice and Moriconi (2005) and Moller and Steffensen (2007)) restrict themselves to single-premium contracts and neglect mortality to simplify the presentation or to obtain analytical solutions. However, in the presence of surrender, generalizations which include periodic premiums and mortality risk are not always straightforward; see, e.g., Bacinello (2005).

In this paper, we develop a general model framework for the ALM of life insurance products. The complexity of the model is chosen such that, on the one hand, most of the models previously proposed in the literature and the most important features of life insurance product management are included. As a consequence, closed-form solutions will only be available in special cases. On the other hand, the model is supposed to remain transparent and modular, and it should be possible to simulate the model efficiently. Therefore, we use a discrete time framework in which all terms can be derived easily and can be computed recursively. We use a stochastic two-factor model to simulate the behaviour of the capital markets, while the development of the biometric parameters is assumed to be deterministic. The asset allocation is dynamic with the goal of keeping a certain percentage of stocks. The bonus declaration mechanism is based on the reserve situation of the company as proposed in Grosen and Jorgensen (2000). Surrender is modelled and analysed using experience-based surrender tables. Different life insurance product specifics are incorporated via premium, benefit and surrender characteristics in a fairly general framework. In contrast to most of the existing literature, where only the valuation or the development of a single policy is considered, we model the development of a portfolio of policies using model points. Each model point corresponds to an individual policyholder account or to a pool of similar policyholder accounts which can be used to reduce the computational complexity, in particular in the case of very large insurance portfolios. Thus we can also investigate effects which arise from the pooling of non-homogeneous contracts, as in Hansen and Miltersen (2002), where the pooling of two contracts is considered.

The outline of this paper is as follows. In Section 2, we start with the main layout of the balance sheet. Then, in Section 3, the capital market model is described. In Section 4, management rules regarding the capital allocation, the bonus declaration and the shareholder participation are defined. The specification of the insurance products and the individual policyholder accounts is subject of Section 5. In Section 6, the future development of the balance sheet items introduced in Section 2 is derived. Numerical results for example portfolios and model parameters are shown in Section 7. Here, we illustrate the influence and the interaction of the parameters of the model by sensitivity analyses and iso-default probability curves. We particularly investigate the impact of mortality and surrender on the default probabilities of the insurance company. The paper closes in Section 8 with concluding remarks.

Table 1
Simplified balance sheet for a portfolio of life insurance policies

Assets		Liabilities	
Capital	C_k	Actuarial reserve	D_k
		Allocated bonus	B_k
		Free reserve	F_k
		Equity	Q_k

2. Balance sheet model

The main focus of our model is on simulating the temporal development of the most important balance sheet items for a portfolio of insurance policies. In this section, we indicate the overall structure of the balance sheet. The determination of the single balance sheet items and the modelling of their future development is the subject of the following sections.

We model all terms in discrete time. Here, we denote the start of the simulation by time $t = 0$ and the end of the simulation by $t = T$ (in years). The interval $[0, T]$ is decomposed into K periods $[t_{k-1}, t_k]$, $k = 1, \ldots, K$ with $t_k = k\,\Delta$. Throughout this paper, the period length $\Delta t = T/K$ is equal to one month, which is in line with conventions for insurance contract sales.[1] The balance sheet items at time t_k, $k = 0, \ldots, K$, used in our model are shown in Table 1.

The asset side consists of the market value C_k of the company's assets at time t_k. On the liability side, the first item is the book value of the actuarial reserve D_k, i.e., the

[1] Shorter or longer period lengths can be realized in a straightforward way, though.