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Loss analysis of a life insurance company applying discrete-time risk-minimizing hedging strategies

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Abstract

The present paper investigates the net loss of a life insurance company issuing equity-linked pure endowments in the case of periodic premiums. Due to the untradability of the insurance risk which affects both the in- and outflow side of the company, the issued insurance claims cannot be hedged perfectly. Furthermore, we consider an additional source of incompleteness caused by trading restrictions, because in reality the hedging of the contingent claims is more likely to occur at discrete times. Based on Møller [Møller, T., 1998. Risk-minimizing hedging strategies for unit-linked life insurance contracts. Astin Bull. 28, 17–47], we particularly examine the situation, where the company applies a time-discretized risk-minimizing hedging strategy. Through an illustrative example, we observe numerically that only a relatively small reduction in ruin probabilities is achieved with the use of the discretized originally risk-minimizing strategy because of the accumulated extra duplication errors caused by discretizing. However, the simulated results are highly improved if the hedging model instead of the hedging strategy is discretized. For this purpose, Møller's [Møller, T., 2001. Hedging equity-linked life insurance contracts. North Amer. Actuarial J. 5 (2), 79–95] discrete-time (binomial) risk-minimizing strategy is adopted.

JEL classification: G10; G13; G22

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1. Introduction

The topic of insolvency risk of life insurance companies has attracted a great deal of attention. Since the 1980s, a long list of defaulting life insurance companies in Europe, Japan and USA has been reported. Here are two noticeable examples from the United States: First Executive Life Insurance Co. in 1991, the 12th largest bankruptcy in the United States in the period 1980–2005, and Conseco Inc. in 2002, the 3rd largest bankruptcy in the United States in the same period.[1] In Japan, the following life insurance carriers defaulted: Nissan Mutual Life in 1997, Chiyoda Mutual Life Insurance Co. and Kyoei Life Insurance Co. in 2000 and Tokyo Mutual Life Insurance in 2001. In Europe, there were the following most noticeable insolvency cases: Garantie Mutuelle des Fonctionnaires in France in 1993, the world's oldest life insurer Equitable Life in the United Kingdom in 2000 and Mannheimer Leben in Germany in 2003. Therefore, the task of how to reduce insolvency risk becomes a more and more important topic.

The insolvency risk of an insurance company can usually be reduced in two different ways: externally or internally. Concerning external risk management, a regulator may be introduced who imposes an intervention rule in order to prevent the insurance company from insolvency. This is the approach taken e.g. by Grosen and Jørgensen (2002).[2] In

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[1] Data were taken from http://www.bankruptcydata.com.

[2] Bernard et al. (2005) extend Grosen and Jørgensen by incorporating the stochastic interest rate. Chen and Suchanecki (2007) extend Grosen and Jørgensen by allowing for more general bankruptcy procedures.

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their model, the firm defaults and is liquidated if up to the maturity time the value of the total assets has not been sufficiently high to cover the nominal liability multiplied by some pre-specified constant parameter. The regulator controls the strictness of the intervention by setting the size of this parameter. Concerning internal risk management, the insurance company actively manages its exposure to insolvency by appropriately hedging the risks of the issued contracts. This approach has already been used e.g. in Mahayni and Schlögl (2003). They mainly investigate how to determine the contract parameters conservatively and implement robust risk management strategies. It is worth mentioning that different contract designs and different hedging criteria would lead to very different results if using this approach. In the present paper, we mainly study the case when the insurance company applies a risk-minimizing hedging strategy to equity-linked pure endowments. Moreover, we go one step further and investigate the net loss of the contract-issuing company.

Equity-linked life insurance contracts are an example of the interplay between insurance and finance. A rapidly increasing volume of equity-linked life insurance contracts is observed. Neglecting some detailed subtleties, many life insurance products provided outside Europe have similar features to equity-linked life insurance, although they have completely different names, like segregated fund contracts in Canada which have become a popular alternative to mutual fund investment, unit-linked insurance products in United Kingdom and variable annuities in United States which are similar to segregated funds. A detailed description of the history of equity-linked life insurance and diverse contract forms can be found in e.g. Hardy (2003). In contrast to the financial risks, the insurance risk is not tradable in the financial market. There are different methods to deal with this untradable risk. Following Brennan and Schwartz (1979), most authors (e.g. Bacinello and Ortu (1993), Bacinello and Persson (2002), and Miltersen and Persson (2003)) replace the uncertainty of the insured individuals' death/survival by the expected values according to the law of large numbers. So, the actual insurance claims, including mortality risk as well as financial risk, are replaced by modified claims, which only contain financial uncertainty. This allows the use of standard financial valuation and hedging techniques for complete markets. Although some other authors add mortality risk to their model, they neglect the hedging perspective and mainly deal with the fair valuation of the equity-linked life insurance contracts, see e.g. Aase and Persson (1994), Ekern and Persson (1996) and Nielsen and Sandmann (1995, 1996, 2002). In contrast to all the authors mentioned above, Møller (1998) attempts to **hedge** the combined actuarial and financial risk. In his work, continuously adjustable risk-minimizing (in the sense of variance-minimizing) hedging strategies are determined for equity-linked life insurance contracts. In this paper, we use Møller's risk-minimizing strategy with a modification, namely a trading restriction is imposed on this continuous strategy, i.e., the hedging of the contingent claim occurs at discrete times only. Therefore, the considered model is incomplete in two aspects where the incompleteness results not only from the mortality risk but also from the trading restriction.

Through an illustrative simulation example, it is observed numerically that a substantial reduction in the ruin probability[3] is achieved by using the time-discretized risk-minimizing strategy, in comparison with the scenario where the insurer invests the premiums in a risk free asset with a rate of return corresponding to the market interest rate. However, the extent of the reduction becomes less apparent and the advantage of using this strategy almost disappears when the trading frequency is increased. This is due to the fact that extra duplication errors are caused when the original mean-self-financing risk-minimizing hedging strategy is discretized with respect to time, and because these errors increase with the frequency. In order to improve the numerical results, another type of discrete-time risk-minimizing strategy is taken into consideration. It is obtained by discretizing the hedging model instead of the hedging strategy. For this purpose, we consider the Cox-Ross-Rubinstein (1979) model (CRR), which converges in the limit to the Black and Scholes' (1973) model. In this discrete-time framework, the binomial risk-minimizing strategy in Møller (2001) is adopted. When comparing the simulation results with the scenario where the strategy is discretized, we observe considerably smaller ruin probabilities, in particular, when the frequency is increased.

This paper is organized as follows: In Section 2, the net loss of an insurance company is defined, and for two simple scenarios the loss is computed. Section 3 focuses on the net loss caused by using the time-discretized originally continuous risk-minimizing hedging strategies. Section 4 demonstrates how to calculate the relevant discretized risk-minimizing strategy with the help of an example. In Section 5, by simulating the ruin probability caused by discretizing the hedging strategy, we show that some of the numerical results are not very satisfactory. In Section 6, the hedging model is discretized instead of the hedging strategy, and the numerical results are improved substantially. Section 7 concludes the paper.

2. Net loss and two extreme cases

This section aims at introducing the model setup and specifying the underlying financial market and the loss function. We consider an insurance company operating on the time horizon $[0, T]$. Suppose, at time 0, n identical customers of age x close the same pure endowment with the insurance company, which promises each of them a payment of $f(T, S)$ at T if they survive the maturity date. The function $f(T, S)$ describes the dependence of the final payment on the evolution of the stock price. It can be a function of the terminal stock price S_T only or of the whole path of the stock, and possibly it contains embedded options. In return, each customer pays a premium of K periodically, which is determined at the beginning of the contract and which will be kept constant through the duration of the contract. In other words, as long as the customer is alive, the customer pays K at each $t \in$

[3] Due to the specific modelling of the contract (pure endowments), the ruin probability equals the relative frequency the simulated net loss of the insurer at the maturity of the contract is larger than zero.