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Risk-neutral valuation of participating life insurance contracts in a stochastic interest rate environment☆

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Abstract

Over the last years, the valuation of life insurance contracts using concepts from financial mathematics has become a popular research area for actuaries as well as financial economists. In particular, several methods have been proposed of how to model and price *participating* policies, which are characterized by an annual interest rate guarantee and some bonus distribution rules. However, despite the long terms of life insurance products, most valuation models allowing for sophisticated bonus distribution rules and the inclusion of frequently offered options assume a simple Black–Scholes setup and, more specifically, deterministic or even constant interest rates.

We present a framework in which participating life insurance contracts including predominant kinds of guarantees and options can be valuated and analyzed in a stochastic interest rate environment. In particular, the different option elements can be priced and analyzed separately. We use Monte Carlo and discretization methods to derive the respective values.

The sensitivity of the contract and guarantee values with respect to multiple parameters is studied using the bonus distribution schemes as introduced in [Bauer, D., Kiesel, R., Kling, A., Ruß, J., 2006. Risk-neutral valuation of participating life insurance contracts. Insurance: Math. Econom. 39, 171–183]. Surprisingly, even though the value of the contract as a whole is only moderately affected by the stochasticity of the short rate of interest, the value of the different embedded options is altered considerably in comparison to the value under constant interest rates. Furthermore, using a simplified asset portfolio and empirical parameter estimations, we show that the proportion of stock within the insurer's asset portfolio substantially affects the value of the contract.
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1. Introduction

Participating life insurance contracts are characterized by an interest rate guarantee and some bonus distribution rules, which provide the possibility for the policyholder to *participate* in the earnings of the insurance company. While in England or other Anglo-Saxon countries the interest rate guarantee is often given on a point-to-point basis, the predominant kinds of insurance contracts in other markets as, e.g., the German market include the so-called cliquet style guarantees. Within such products, a certain guaranteed rate of return plus some surplus is credited to the policyholder's account each year. Furthermore, these contracts often contain other option features such as a surrender option.

The analysis of participating life insurance contracts with a minimum interest rate requires a realistic model of bonus payments. Grosen and Jørgensen (2000) establish some general principles for modeling bonus schemes: They argue that life

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insurance policies should provide a low-risk, stable and yet competitive investment opportunity. In particular, the surplus distribution should reflect the so-called "*average interest principle*", which states that insurers are to build up reserves in years of high returns and use the accumulated reserves to keep the surplus stable in years with low returns without jeopardizing the company's solvency. Aside from an interest rate guarantee and a distribution mechanism for excessive returns which suits these principles, the model of Grosen and Jørgensen (2000) further includes the possibility for the insured to surrender. In this case, the policyholder obtains the account value whereas the reserves remain with the company. As no closed-form solution for the policy value can be derived, Monte Carlo methods are used for the valuation of the contract.

In Jensen et al. (2001), the same valuation problem is tackled in an alternative way: Within each period, they show that the value function follows a known Partial Differential Equation (PDE), namely the Black–Scholes PDE, which can be solved using finite difference methods. At inception of each period, arbitrage arguments ensure the continuity of the value function. Based on these insights, they derive a backward iteration scheme for pricing the contract. This approach is extended and generalized in Tanskanen and Lukkarinen (2003). In particular, they do not use finite differences in order to solve the PDE, but derive an integral solution which is based on the transformation of the Black–Scholes PDE into a one-dimensional heat equation. Their model permits multiple distribution mechanisms including the one from Grosen and Jørgensen (2000).

The distribution mechanisms considered in Bauer et al. (2006) also satisfy the general principles provided in Grosen and Jørgensen (2000). Additionally, their general framework allows for payments to the shareholders of the company as a compensation for the adopted risk. Furthermore, Bauer et al. (2006) show how the value of a contract can be separated into its single components and derive an equilibrium condition for a fair contract.

However, all these contributions perform the valuation in a simple Black–Scholes model for the financial market and, in particular, assume deterministic or even constant interest rates. Considering the long terms of insurance products, this assumption does not seem adequate. In contrast, other publications allow for a stochastic evolution of interest rates. However, these articles consider point-to-point guarantees rather than cliquet style guarantees (see, e.g., Barbarin and Devolder (2005), Bernard et al. (2005), Briys and de Varenne (1997), or Nielsen and Sandmann (1995)), or do not allow for the consideration of typical distribution schemes or option features embedded in many life insurance contracts (see, e.g., De Felice and Moriconi (2005) or Miltersen and Persson (1999)).

The present paper fills this gap: We adopt the methodology presented in Bauer et al. (2006) and incorporate more consistent models for the behavior of interest rates into their model. In order to take into account all typical components of a participating life insurance contract, different numerical methods are presented. Besides Monte Carlo methods, we present a discretization approach based on the numerical solution of certain PDEs, which allows us to consider the non-European surrender option. We study the impact of various parameters on the contract value focusing on the parameters which emerge due to the stochasticity of the evolution of interest rates. Furthermore, using a simplified asset portfolio and empirical parameter estimations, we study the impact of the proportion of stock within the insurer's asset portfolio on the contract value.

The remainder of the paper is organized as follows: In Sections 2 and 3 we briefly introduce the model and valuation methodology from Bauer et al. (2006), respectively. Section 4 presents the valuation approaches. In particular, we introduce the two stochastic interest models considered in the paper, namely the well-known models of Vasiček (1977) and Cox et al. (1985) for the evolution of the short rate, and explain our valuation algorithms as well as their implementation. Our results are presented in Sections 5 and 6. Besides the values of the contract and the embedded options, we examine their sensitivities toward changes in several parameters and give economic interpretations. While in Section 5 we focus on parameters that come into play due to the stochasticity of the interest rates, Section 6 is devoted to the study of the impact of the proportion of stock within the insurer's asset portfolio. Section 7 closes with a summary of the main results and an outlook for future research.

Table 1
Simplified balance sheet

Assets	Liabilities
A_t	L_t R_t
A_t	A_t

2. Model

We use the simplified balance sheet given in Table 1 to model the insurance company's financial situation. Here, A_t denotes the market value of the insurer's asset portfolio at time t, L_t the policyholders' account balance at time t, and $R_t = A_t - L_t$ the time t reserve. We further assume that dividend payments d_t to the shareholders occur at the policy anniversaries $t \in \{1, \ldots, T\}$. We consider only one simple life insurance contract, namely a single premium ($P = L_0$) term-fix insurance contract issued at time 0 and maturing after T years ignoring any charges. Given this contract, the benefit does not depend on biometric circumstances, but merely on the development of the insurer's assets and liabilities. Thus, at maturity T the policyholder or the policyholder's heirs receive $P \frac{L_T}{L_0} \overset{P=L_0}{=} L_T$.

2.1. Surplus distribution schemes

The question of how the surplus is distributed to the policyholders in practice is highly delicate and demands legal as well as strategic considerations within the insurance company. Our general model allows for any management decision made