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Combining fair pricing and capital requirements for non-life insurance companies

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ABSTRACT

The aim of this article is to identify fair equity-premium combinations for non-life insurers that satisfy solvency capital requirements imposed by regulatory authorities. In particular, we compare target capital derived using the value at risk concept as planned for Solvency II in the European Union with the tail value at risk concept as required by the Swiss Solvency Test. The model framework uses Merton's jump-diffusion process for the market value of liabilities and a geometric Brownian motion for the asset process; fair valuation is conducted using option pricing theory. We show that even if regulatory requirements are satisfied under different risk measures and parameterizations, the associated costs of insolvency – measured with the insurer's default put option value – can differ substantially.

1. Introduction

Recent developments in Europe include new solvency capital requirements that are based on the market value of assets and liabilities (Solvency II in the European Union; the Swiss Solvency Test). Insurance companies must ensure that their available economic capital suffices to cover the required solvency capital. In addition, competitive conditions in the insurance and capital markets should lead to equity-premium combinations that provide a net present value of zero for equityholders and policyholders. In this paper, we identify minimum safety levels using the default put option value for fair equity-premium combinations that simultaneously satisfy solvency capital requirements. In this setting, fair pricing is conducted using option pricing theory; solvency capital requirements are compared using the "value at risk" (Solvency II) and the "tail value at risk" (Swiss Solvency Test) approaches.

In the literature on property-liability insurance, option pricing theory is employed for pricing insurance contracts and default risk in, e.g., Merton (1977), Doherty and Garven (1986), Cummins (1988), Cummins and Sommer (1996), as well as D'Arcy and Dyer (1997). Recently, this approach has been used for capital allocation purposes, by, for example, Myers and Read (2001), Sherris (2006), Sherris and van der Hoek (2006), Gründl and Schmeiser (2007), and Yow and Sherris (2007). Based on Fairley (1979), Taylor (1995) and Sherris (2003) use an equilibrium framework in order to examine the interaction between capitalization of an insurer and rate making. General capital requirements under different model assumptions and safety levels are discussed in, e.g., Rytgaard and Savelli (2004); solvency capital calculations regarding the Swiss Solvency Test (SST) are presented in detail in Luder (2005).

The literature generally focuses on fair pricing, capital structure, or solvency requirements. However, apart from rate making and capitalization (Taylor, 1995; Sherris, 2003), these aspects are usually studied individually. In this paper, we add to the literature by combining fair pricing and solvency capital requirements to gain a better understanding of the effect of solvency regulation on the cost of insolvency (measured with the default put option value). The framework for the property-liability insurance company is based on Doherty and Garven (1986). The model incorporates corporate taxation and the risk of insolvency. In contrast to the setting in Doherty and Garven (1986), the model framework in this paper is extended by using Merton's jump-diffusion process for the market value of liabilities and a geometric Brownian motion for the asset process. In numerical analyses, we first calculate fair equity-premium combinations for a given safety level (measured with the default put option value). Then, for the obtained capital structure, the target capital requirements according to Solvency II and SST are contrasted with the available economic capital. In addition, shortfall probabilities are also provided.

The remainder of the paper is organized as follows. Section 2 describes the model framework of the property-liability insurer. Section 3 discusses concepts for solvency capital requirements

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under SST and Solvency II. Numerical results, including a sensitivity analysis, are contained in Section 4. Finally, Section 5 concludes.

2. Model framework

2.1. A contingent claims approach under corporate taxes

In the analysis, we employ a simplified model of a property-liability insurer based on Doherty and Garven (1986) as well as Gatzert and Schmeiser (2008). In this one-period setting, policyholders and equityholders make an initial contribution of P_0^τ (premium payments) and E_0^τ (equity capital), respectively, where τ denotes the corporate tax level. The total value $P_0^\tau + E_0^\tau = A_0^\tau$ is then invested in a reference portfolio.

At time $t = 1$, policyholders receive payment for incurred losses L_1 if the insurer is solvent; equityholders obtain the remainder of A_1^τ. In the case of insolvency, the assets A_1^τ are fully distributed to policyholders, and equityholders receive nothing.[1] Hence, the payoff to the policyholders is formally given by

$$P_1^\tau = L_1 - \max(L_1 - A_1^\tau, 0) + T_1,$$

while the equityholders receive,

$$E_1^\tau = \max(A_1^\tau - L_1, 0) - T_1.$$

Here, T_1 denotes the tax claim for a corporate tax level of τ with

$$T_1 = \tau \cdot \max\left((E_0^\tau + P_0^\tau)\left(\frac{A_1^\tau}{A_0^\tau} - 1\right) + P_0^\tau - L_1, 0\right).$$

Thus, the government collects corporate taxes on the insurer's investment income and underwriting profits.[2] Fair valuation of claims is conducted using risk-neutral valuation. The expected present value of the policyholders' claim Π^P is then given by the expected payoff (under the risk-neutral measure $\mathbb{Q}$) in $t = 1$ discounted with the (continuous) risk-free interest rate r, leading to

$$\begin{aligned}\Pi^P &= E^{\mathbb{Q}}(\exp(-r)P_1^\tau)\\ &= E^{\mathbb{Q}}(\exp(-r)L_1) - E^{\mathbb{Q}}(\exp(-r)\max(L_1 - A_1^\tau, 0))\\ &\quad + E^{\mathbb{Q}}(\exp(-r)T_1)\\ &= \Pi^L - \Pi^{DPO} + \Pi^{T_1}. \end{aligned} \quad (1)$$

Eq. (1) shows that Π^P consists of the present value of losses $\Pi^L (=L_0)$ less the default put option (DPO) value Π^{DPO} and the present value of tax payments Π^{T_1}. The present value of the payoff to the equityholders Π^E is given by

$$\Pi^E = E^{\mathbb{Q}}(\exp(-r)E_1^\tau) = E^{\mathbb{Q}}(\exp(-r)\max(A_1^\tau - L_1, 0)) - \Pi^{T_1}. \quad (2)$$

To create a fair situation, the net present value must be zero, which corresponds to the value of the payoffs being equal to the initial contributions:

$$\Pi^P = P_0^\tau, \quad (3)$$

$$\Pi^E = E_0^\tau. \quad (4)$$

Solving the fairness conditions in Eq. (3) – or (4) – leads to an infinite number of initial equity-premium combinations with different DPO values that are fair for both policyholders and equityholders. Gatzert and Schmeiser (2008) show that a fixed initial asset value A_0 implies a fixed DPO value and thus a fixed safety level for the insurer before and after taxation. Therefore, if

$$A_0 = A_0^\tau = E_0^\tau + P_0^\tau, \quad (5)$$

then Π^{DPO} remains unchanged for different corporate tax rates τ.

2.2. Modeling assets and liabilities

For the asset model, we use a geometric Brownian motion; the market value of liabilities is modeled using a jump-diffusion process, as suggested by Merton (1976).[3] Under the real-world measure $\mathbb{P}$, the asset process is described by

$$dA_t = \mu_A A_t dt + \sigma_A A_t dW_{A,t}^{\mathbb{P}};$$

the liability process evolves as

$$\frac{dL_t}{L_{t-}} = \mu_L dt + \sigma_L dW_{L,t}^{\mathbb{P}} + dJ_t,$$

with μ and σ denoting the drift and volatility of the stochastic processes and $L_{t-} = \lim_{u \uparrow t} L_u$. $W_A^{\mathbb{P}}$ and $W_L^{\mathbb{P}}$ are standard $\mathbb{P}$-Brownian motions. The two Brownian motions are correlated with

$$dW_A dW_L = \rho dt.$$

Furthermore, J is a process that is independent of W with piecewise constant sample paths and can be represented as

$$J_t = \sum_{j=1}^{N_t}(Y_j - 1).$$

Thus, if the jth jump occurs at time t, liabilities jump from L_{t-} to $L_{t-}Y_j$, where $Y_j - 1$ is the size of the jump. N_t is a Poisson process counting the number of jumps by time t with intensity λ (the average number of jumps per year). In this setting, N, W, and Y_j are stochastically independent. Solutions of the stochastic differential equations above are given by (see, e.g., Merton, 1976)

$$A_t = A_0 \cdot \exp\left((\mu_A - \sigma_A^2/2)t + \sigma_A W_{A,t}^{\mathbb{P}}\right)$$

and

$$L_t = L_0 \cdot \exp\left((\mu_L - \sigma_L^2/2)t + \sigma_L W_{L,t}^{\mathbb{P}}\right) \cdot \prod_{j=1}^{N_t} Y_j.$$

In the Merton model, jumps are interpreted as idiosyncratic shocks that concern the individual company and only marginally affect the economy as a whole. Therefore, insurance jump risk is considered to be nonsystematic and diversifiable; thus no risk premium is required by the market.[4] Hence, changing the real-world measure $\mathbb{P}$ to the equivalent risk-neutral martingale measure $\mathbb{Q}$ leads to

[1] This model assumes competitive insurance markets. Imperfect market competition is implemented by, e.g., Yow and Sherris (2007) in the context of performance measurement of value at risk methods used for capital allocation and incorporation of costs of capital into insurance pricing for a multi-line property and liability insurer.

[2] In this setting, the tax burden is carried solely by the policyholders. The consideration is caused by double-taxation, i.e., equityholders could also directly invest in financial assets (e.g., government bonds), where only taxes on the individual level are raised and hence no corporate taxes are charged. In this case, equityholders would not agree to carry additional (i.e., corporate) taxation by investing in an insurance company (see also, e.g., Doherty and Garven, 1986; Gründl and Schmeiser, 2007). In this context, it is implicitly assumed that there is still demand for insurance coverage given the premium principle in Eq. (1). A further discussion regarding the demand of insurance in the case of frictional costs and its interaction to shareholder value is provided in Yow and Sherris (2007, pp. 40–42).

[3] Merton's jump-diffusion model has been applied to insurance liabilities by, e.g., Cummins (1988).

[4] In general, including jumps in the liability process leads to an incomplete market with infinitely many martingale measures. Thus, without the assumption that jump risk is nonsystematic, unique prices for claims cannot be derived when using no-arbitrage pricing. As an alternative to the Merton assumption, one can proceed as is done in, e.g., Møller (2004), choose a martingale measure (e.g., based on risk preferences of the investor, mean-variance hedging, risk-minimization), and then conduct the pricing under this measure.