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Applications of a multi-state risk factor/mortality model in life insurance

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ABSTRACT

Mortality rates are known to depend on socio-economic and behavioral risk factors, and actuarial calculations for life insurance policies usually reflect this. It is typically assumed, however, that these risk factors are observed only at policy issue, and the impact of changes that occur later is not considered. In this paper, we present a discrete-time, multi-state model for risk factor changes and mortality. It allows one to more accurately describe mortality dynamics and quantify variability in mortality. This model is extended to reflect health status and then used to analyze the impact of selective lapsation of life insurance policies and to predict mortality under reentry term insurance.

1. Introduction

A thorough understanding of human mortality is crucial in life insurance product development, pricing, valuation and profitability analysis. More specifically, life insurance actuaries must understand insured lives mortality and how it is influenced by various policyholder risk factors and product characteristics. This is essential in creating fair pricing structures and in accurately evaluating the risk associated with a life insurance product.

Mortality assumptions typically involve a collection of age- and policy duration-specific mortality rates for each risk class, and risk classes may differ based on sex, smoking status, and possibly other relevant health or socio-economic information that is known at policy issue. Therefore, these mortality rates are intended to represent the average rates of death for insured lives who were in a given risk class when their policy was issued, and these rates do not contemplate changes in risk classification or health variables that may occur after policy issue. In the absence of further information that may become available later, the mortality rates are the best prediction available at the time of policy issue. However, such mortality assumptions are limited in their ability to appropriately predict variability in mortality experience (i.e. mortality risk) and their ability to represent the underlying mortality dynamics.

As an alternative, one could allow mortality assumptions to reflect knowledge of how various mortality risk factors change over time. Mortality risk factors are important, as they influence an insured's probability of entering various health states for which the rate of death is elevated. These risk factors, both behavioral and socio-economic, can change over time. From the insurer's perspective, these changes can be viewed as random. Thus, it is appropriate to consider a stochastic model that describes changes in risk factors as well as mortality.

Tolley and Manton (1991) proposed a Markov model to calculate the expected cost for health insurance reflecting future changes in health condition. The impact of changes in several health indicators on the occurrence of diseases covered by critical illness insurance is investigated in Macdonald et al. (2005a,b). The authors used a continuous-time Markov model.

Kwon and Jones (2006) proposed and explored a discrete-time Markov chain model for mortality. The model allows mortality rates to depend on four behavioral/socio-economic risk factors: smoking, physical activity, marital status, and income. To keep the model simple, each risk factor is assumed to have just two levels, resulting in 16 risk factor states. Including the "dead" state resulted in a 17 state model. Complete specification of the model requires age specific probabilities of changes in risk factor state and death. These probabilities were estimated using data from the Canadian National Population Health Survey. This model enables one to analyze the mortality risk in life insurance in a way that allows for random risk factor changes after policy issue.

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The purpose of this paper is to illustrate the use of such a multi-state model in two interesting analyses involving life insurance. The first explores the impact of selective lapsation. This occurs when there is a greater tendency for healthy lives to lapse their life insurance policies than for unhealthy lives to do so. The idea here is that the unhealthy lives have a greater perceived need for the insurance. The second analysis relates to reentry term insurance, where insureds have an option at the end of each term of a renewable term policy to provide updated evidence of insurability in order to receive "select" premium rates. If an insured chooses not to provide such evidence or if an insured is deemed not to qualify for the select rates, his/her premium rates will increase to those appropriate to a less healthy group. The policy may specify the premium rates that will be charged in this case, or it may specify the maximum rates that may be charged.

In order to accomplish the analyses of selective lapsation and the reentry term option, we require an extension of the model of Kwon and Jones (2006). While the different risk factor states in this model are associated with different mortality, they do not capture the very significant differences needed to affect decisions by the insured about whether to continue or lapse a policy or decisions by the insurer about whether or not the insured still qualifies for select premium rates. Our model extension involves differentiating between "healthy" lives and "unhealthy" lives. Those in the former group are more likely to lapse their policies, and would qualify for select rates. Those in the latter group are less likely to lapse their policies and would not qualify for select rates. The definitions of "healthy" and "unhealthy" are rather vague, and though an insured is healthy at policy issue, changes in this status are generally unobservable from the insurer's perspective. However, with few assumptions, we are able to achieve the desired model extension.

The remainder of the paper is organized as follows. In Section 2, we present the model of Kwon and Jones (2006). This is followed by the development of the extended model in Section 3. Then, in Sections 4 and 5, we use the extended model to analyze selective lapsation and reentry term insurance. Section 6 closes the paper with some concluding remarks.

2. A multi-state risk factor/mortality model

In the model developed by Kwon and Jones (2006), four important mortality risk factors in addition to age, denoted by x, and sex, denoted by Z_{sex} (1 if female, 0 if male), were identified. This was based on an analysis of data from the (Canadian) National Population Health Survey (NPHS). These risk factors are described below.

Income adequacy

Income adequacy is based on total household income and size of family. Adequate income was defined (by the NPHS) as income levels which are \$30,000 or more for a household with 1 or 2 persons, \$40,000 or more for a household with 3 or 4 persons, or \$60,000 or more for a household with 5 persons or more. Let

$$Z_{\text{income}} = \begin{cases} 1 & \text{if adequate income} \\ 0 & \text{if inadequate income.} \end{cases}$$

Marital status

The "married" category includes persons who are married, living common-law or have a partner. Let

$$Z_{\text{marriage}} = \begin{cases} 1 & \text{if married} \\ 0 & \text{if single.} \end{cases}$$

Table B.1
States in the mortality model

State j	Z_{income}	Z_{marriage}	Z_{smoking}	Z_{activity}
1	0	0	0	0
2	1	0	0	0
3	0	1	0	0
4	1	1	0	0
5	0	0	1	0
6	1	0	1	0
7	0	1	1	0
8	1	1	1	0
9	0	0	0	1
10	1	0	0	1
11	0	1	0	1
12	1	1	0	1
13	0	0	1	1
14	1	0	1	1
15	0	1	1	1
16	1	1	1	1
17	Dead			

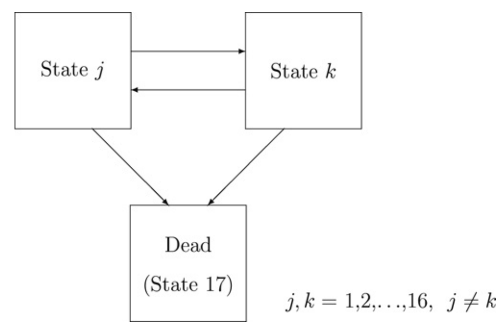

Fig. A.1. Risk factor model.

Smoking status

Smoking status indicates whether or not the person is a current smoker; it does not take into account smoking history or intensity of smoking. Let

$$Z_{\text{smoking}} = \begin{cases} 1 & \text{if current smoker} \\ 0 & \text{if non-smoker.} \end{cases}$$

Physical activity

This variable is based on a physical activity index derived from energy expenditure values which were estimated based on the response to several questions on the NPHS questionnaire. The amount of exercise required to be in the "active" category is approximately that required for health benefits. Let

$$Z_{\text{activity}} = \begin{cases} 1 & \text{if physically active} \\ 0 & \text{if inactive.} \end{cases}$$

Sixteen risk factor states are defined by the combinations of income adequacy, marital status, smoking status and physical activity. A seventeenth state is used to indicate death. The risk factor combination for each state is shown in Table B.1.

Kwon and Jones (2006) assumed that individuals move among the 17 states according to a nonhomogeneous discrete-time Markov chain. The setup is illustrated in Fig. A.1. The transition matrix for a given age and sex is denoted by

$$Q(x, Z_{\text{sex}}) = \begin{pmatrix} Q_{1,1}(x, Z_{\text{sex}}) & Q_{1,2}(x, Z_{\text{sex}}) & \ldots & Q_{1,17}(x, Z_{\text{sex}}) \\ Q_{2,1}(x, Z_{\text{sex}}) & Q_{2,2}(x, Z_{\text{sex}}) & \ldots & Q_{2,17}(x, Z_{\text{sex}}) \\ \vdots & \vdots & \ddots & \vdots \\ Q_{17,1}(x, Z_{\text{sex}}) & Q_{17,2}(x, Z_{\text{sex}}) & \ldots & Q_{17,17}(x, Z_{\text{sex}}) \end{pmatrix},$$