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Valuation and hedging of participating life-insurance policies under management discretion

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ABSTRACT

The valuation and hedging of participating life insurance policies, also known as with-profits policies, is considered. Such policies can be seen as European path-dependent contingent claims whose underlying security is the investment portfolio of the insurance company that sold the policy. The fair valuation of these policies is studied under the assumption that the insurance company has the right to modify the investment strategy of the underlying portfolio at any time. Furthermore, it is assumed that the issuer of the policy does not setup a separate portfolio to hedge the risk associated with the policy. Instead, the issuer will use its discretion about the investment strategy of the underlying portfolio to hedge shortfall risks. In that sense, the insurer's investment portfolio serves simultaneously as the underlying security and as the hedge portfolio. This means that the hedging problem can not be separated from the valuation problem. We investigate the relationship between risk-neutral valuation and hedging of these policies in complete and incomplete financial markets.

1. Introduction and literature review

Participating life insurance contracts, also called with-profits contracts, have been issued over the past decades by many insurance companies throughout the world. Although the details of the contracts vary significantly between insurer's there are some common features that all with-profits contracts share. An overview about common forms of with-profits contracts in Europe and the United States is provided by Cummins et al. (2004). We will consider with-profits contracts which are typical in the UK.

In March 2002 the Financial Services Authority in the UK published an issue paper related to the with-profits Review undertaken by the FSA (2002) in which some of the important features of with-profits contracts are summarized. In particular, they say that:

> "With-profits products are sold as long duration products and have certain features which normally include:
>
> - policyholder premiums are held in a pooled fund that is invested in a range of assets, a significant proportion of which are usually in the form of equities and property;
> - certain guarantees, which usually increase over the lifetime of the policy. For example, the payment of a guaranteed amount at maturity or retirement, or on death. The guaranteed amount may build through the duration of the contract by the addition of regular bonuses. A final bonus, which does not form part of this guaranteed amount, may be added at the end of the contract;"

The FSA mentions a number of other features (including smoothing of guarantees and allowing the pooled fund to share in profits or losses of the insurers business). For the purpose of this paper we ignore these features.

We want to concentrate on the financial risk arising from the fact that "policyholder premiums are held in a pooled fund that is invested in a range of assets" and, we therefore ignore mortality. Instead, we assume that all contracts reach maturity. Furthermore, we assume that their is only one premium to be paid by the policyholder at the time the contract is issued.

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Since we ignore mortality, buying a with-profits contract can be seen as an investment by the policyholder into the with-profits fund managed by the insurance company, but in contrast to standard investment funds, with-profits contracts provide some protection against low or negative returns. Instead of receiving the final value of the fund, the policyholder receives certain maturity benefits which are guaranteed by the insurer. We assume that the guaranteed maturity benefits increase during the lifetime of the contract and that the rate of increase depends on the performance of the with-profits fund.

In the literature we find different approaches to calculate fair market-consistent values of with-profits contracts and to derive hedging strategies that the insurance company can apply to protect itself against the risk associated with the given guarantees.

In the first approach authors treat the with-profits fund as a fixed investment portfolio, often called the reference portfolio. The price process of the fund is therefore a given stochastic process and the payoff to the policyholder at maturity is a deterministic function of the performance of the with-profits fund. The payoff is therefore a path-dependent European contingent claim. This approach allows for the direct application of methods known from financial mathematics. The first authors to use these methods to price life insurance contracts were Brennan and Schwartz (1976, 1979). These authors considered unit-linked contracts for which the payoff to the policyholder is indeed a contingent claim with a payoff depending on the price of a unit of an externally given reference portfolio. Since then, Market-consistent valuation of participating insurance contracts has been studied by a number of authors. Among these are Persson and Aase (1997), Miltersen and Persson (1999, 2003), and Ballotta (2005). Overviews about the available literature can be found in Kleinow and Willder (2007) and Bauer et al. (2005) and the references therein. A very detailed discussion about the approaches and the results of different authors was carried out by Willder (2004).

In contrast to the assumption of a reference portfolio, a second approach to price and hedge with-profits contracts is based on the assumption that the management of the insurance company has full discretion about the composition of the with-profits fund. In particular, the insurer can use this discretion to reduce or increase the riskiness of the with-profits fund by changing the proportion of money invested into equity shares and the proportion invested into fixed-income securities. In this approach, maturity benefits are still increased according to the performance of the with-profits fund. Since the insurer has control about the with-profits fund any change in the composition of the fund will result in a change of the value of the with-profits insurance contract.

Hibbert and Turnbull (2003) where the first to address this issue. They consider an insurance company in which the management have limited discretion in choosing the assets by applying a fixed rule to increase or decrease the equity exposer of the with-profits fund depending on the value of the insurer's assets and the maturity guarantees already declared. They calculate the fair value of the with-profits contract for these fixed rules.

Kleinow and Willder (2007) consider a more realistic setting by assuming that the management of the insurer has the right to change their investment strategy whenever and however they want to. Any change in this portfolio strategy will lead to a change in the underlying price process that is used to calculate the growth rate of the guaranteed maturity benefits. The insurer is using this discretion to hedge the maturity benefits by making sure that the final value of the with-profits fund is equal to the maturity benefits declared during the lifetime of the contract. In that sense the with-profits fund serves simultaneously as the hedge portfolio and the underlying price process for the payoff at maturity. Kleinow and Willder (2007) have used binomial trees to model the financial market.

There is also literature available on the optimal structure of the insurer's portfolio, see for example Consiglio et al. (2008) and the references therein. This approach takes the discretion of the insurer on the portfolio management into account. However, the focus is on the optimal investment strategy given a certain initial endowment of the insurance company, rather than hedging and pricing the guarantees embedded in the with-profits contract.

The purpose of this paper is to generalize the results by Kleinow and Willder (2007). We will particularly emphasis the relationship between hedging and valuation. Mathematically, we are faced with the following problem. We are given a function H, a random variable Γ and a stochastic process S_0. We want to find a stochastic process V such that its final value $V(T)$ is equal to

$$V(T) = \Gamma \prod_{t=0}^{T-1} H(V(t+1)/V(t))$$

and V/S_0 is a martingale with respect to a given filtration and a family of probability measures. This is a non-standard problem in financial mathematics since the contingent claim $\Gamma \prod_{t=0}^{T-1} H(V(t+1)/V(t))$ is not given but dependents on the "hedge-portfolio" V.

The main contributions of this paper are the introduction of a clear structure of with-profits contracts and a new approach on pricing and hedging these contracts. The proposed definitions of the with-profits contract and the with-profits fund might lack generality but they allow us to concentrate on the use of management discretion for hedging the guarantees. We then show how the guarantees can be hedged without setting up a separate hedge portfolio but choosing an appropriate investment strategy in the with-profits fund. We also show how the hedging and valuation problems are related. In the case of non-hedgeable guarantees, in an incomplete market, we obtain a lower bound for the initial value of the with-profits fund that would allow for super-hedging the guarantees.

We start in Section 2 with setting the scene by introducing the financial market model used in the remainder of the paper. The with-profits contract is then described in detail in Section 3. The pricing and hedging problem is described in Section 4. In Section 5 we discuss the existence of a self-sufficient with-profits fund that is used to solve the hedging and valuation problem, and show some of its properties. We continue in Section 6 with an investigation of the relationship between risk-neutral valuation and hedging. In this section, we also provide a lower bound for the value of a super-hedging strategy in incomplete markets. Finally, we provide an example in Section 7. We conclude the paper and make some remarks about open problems and possible future research in Section 8.

2. The financial market model

Let $(\Omega, \mathcal{F}, \mathrm{P})$ denote a probability space, and let $\mathbb{F} = (\mathcal{F}_t)_{t\in[0,T]}$ for $T \in \mathbb{N}$ be a right-continuous and complete filtration defined on this space. We assume that $\mathcal{F}_0$ is the trivial σ-field.

The tradable assets are a bank account and $d \in \mathbb{N}$ further risky assets. We assume that the value process S_0 of the bank account is adapted to $\mathbb{F}$, $S_0(0) = 1$ and S_0 has finite variation.

The price processes of the d risky assets are denoted by $S_1, \ldots, S_d$. We assume that these are semimartingales with respect to $\mathbb{F}$ under P.

The value of any portfolio consisting of the above assets at any time $t \in [0, T]$ is given by

$$V_\xi(t) = \sum_{i=0}^{d} \xi_i(t) S_i(t) \tag{1}$$

where $\xi = (\xi_0, \xi_1, \ldots, \xi_d)$ is predictable with respect to $\mathbb{F}$. The $(d+1)$-dimensional process ξ is called the portfolio strategy. $\xi_0(t)$