



Optimal asset allocation for a general portfolio of life insurance policies

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ABSTRACT

Asset liability matching remains an important topic in life insurance research. The objective of this paper is to find an optimal asset allocation for a general portfolio of life insurance policies. Using a multi-asset model to investigate the optimal asset allocation of life insurance reserves, this study obtains formulae for the first two moments of the accumulated asset value. These formulae enable the analysis of portfolio problems and a first approximation of optimal investment strategies. This research provides a new perspective for solving both single-period and multiperiod asset allocation problems in application to life insurance policies. The authors obtain an efficient frontier in the case of single-period method; for the multiperiod method, the optimal asset allocation strategies can differ considerably for different portfolio structures.

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1. Introduction

This article attempts to investigate optimal asset allocations in a stochastic investment environment for a general portfolio of life insurance policies. Asset liability matching remains an important issue for long-term liabilities, such as insurance policies or pension funds. Most insurance companies' assets consist of policyholders' premiums, such that each policy represents a specific liability for the insuring firm. Thus, maximizing investment returns may not be the primary goal of insurance companies, which may instead be more concerned with managing policyholders' premiums, such that returns adequately meet future benefits or guarantee profits to policyholders. Besides, insurance policies offered by insurers also vary in duration, so it is not realistic to discuss only the case of asset liability management for a single policy. Therefore, we consider asset liability management in a more general case in which random policy durations exist in a product's profile.

In this paper, we investigate the asset allocation issue on life insurance reserves. Previous research focuses on studying the distribution of reserves, from one policy (e.g. Panjer and Bellhouse, 1980; Bellhouse and Panjer, 1981; Dhaene, 1989) to portfolios (e.g. Waters, 1978; Parker, 1994a,b; Marceau and Gaillardetz, 1999). They examine the distribution of life insurance reserves in a specific interest rate model, for example, AR(1) model or ARCH(1) model. We introduce a multi-asset model and consider the asset allocation problem of life insurance companies.

For the asset allocation issue, widespread investigations consider the investment strategy for a single-period approach

(e.g. Hurlimann, 2002; Sharpe and Tint, 1990; Sherris, 1992, 2006; Wilkie, 1985; Wise, 1984a,b, 1987a,b), whereas multiperiod asset allocations in discrete time rarely have been explored. Extensive research into the optimal multiperiod investment strategy concentrates mostly on continuous time models with dynamic controls (e.g. Chiu and Li, 2006; Emms and Haberman, 2007) or uses the martingale method (e.g. Wang et al., 2007). With these approaches, the optimal strategy takes the new information generated by filtration, but to solve the closed form solution in discrete time, they often suffer from mathematical complexity and intractability. Another approach is to get the numerical solution by stochastic programming (see Dempster, 1980; Carino et al., 1994). It overcomes the disadvantage of finding theoretical solution. The model can be constructed easily and realistically. However, it faces other problems. Stochastic programming constructs the possible asset return scenario by trees. A good description about the market depends on the number of nodes at each decision point. On the other hand, the time cost has an exponential growth as does the number of nodes. Thus, for the purpose of finding the solution in a tolerable time, the node number is often insufficient to describe the real market. Consequently, it is inevitable for the existence of large approximation error. Thus, to examine the appropriate investment strategy in discrete time, we must trade off between the convenience of the method and the accuracy of the solution.

Specifically, we examine two kinds of rebalancing methodologies: constant rebalancing and variable rebalancing. At the beginning of every year, the portfolio mix gets realigned according to one of these two methods. Constant rebalancing means that the portfolio mix should realign to a constant proportion of assets, or the single-period method. Variable rebalancing implies that the portfolio mix realigns with a different proportion of assets each time, or the multiperiod method.

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A continuing business line contains some mature policies and some new policies every year, so the single-period method is more suitable than a multiperiod method, because of the ongoing new policies. If the number of mature policies is close to the number of new policies, the structure of the policy portfolio remains similar between two neighboring decision dates. Therefore, it would be reasonable to adopt a constant rebalance strategy and retain the same weight of assets in a stable proportion every year. If the number of mature policies differs from the number of new policies, the durations of the policy portfolios should differ every year. Therefore, the proportion of constant rebalance requires recalculation, according to the updated durations of policy portfolios every year. Occasionally, a business line ceases to exist, so no new policies occur in the future. In this case, variable rebalancing with a multiperiod method is suitable for matching the rest of the periods of the liabilities.

Because insurance policies typically involve a long duration of more than five years, choosing the optimal investment strategies is crucial to ensure that insurance companies can maximize their profits while reducing their insolvency risk. We propose an optimization approach for analyzing the optimal portfolio problem for both single-period and multiperiod asset allocations.

In turn, we propose an optimization approach to generate the optimal investment strategy of an asset liability management model for long-term endowment policies. The proposed discrete time investment model includes both static, single-period and multiperiod optimal investment strategies for a time-dependent asset return process. We derive the formulae for the first and second moments of the accumulated asset value of the insurer based on a multi-asset return model. With these formulae, we can analyze the portfolio problems for both single-period and multiperiod methods. For the single-period method, we depict an efficient frontier under a constant rebalance strategy, which can be determined from arbitrary policy portfolios. In the case of the multiperiod method, we obtain a first approximation of the optimal asset allocation, as applied to a ceased life insurance product line. The numerical results show that the proportion of cash should increase when we compare a portfolio with uniform years before maturity with a portfolio comprised of new policies.

In Section 2, we formulate the explicit form of the first two moments of accumulated asset value, followed by the mean-variance analysis and an investigation of the optimal asset allocation strategy for various policy portfolios in Section 3. We then examine the parameter sensitivities and discuss the large sample problem in Section 4. Finally, we give the conclusion in Section 5.

2. Model setting

The liability reserve is the value of the difference between the present value of the future benefits and the future premiums received, which is often discounted by conservative rates. Reserves can be viewed as policyholders' credit. The investment manager of an insurance company attempts to make profits from these credits while also ensuring the solvency of the insurer.

The liability reserve of a policy portfolio of an insurer also can be expressed as the present value of the stochastic cash flows (Lai and Fries, 1995; Marceau and Gaillardetz, 1999). Let $CF(j)$ be the cash inflow of the insurer at time j , or the net difference between the premiums received and the benefits paid at time j ($j = 1, \dots, n$). In addition, $v(j)$ is the specified discount factor from time j to time 0. We define L as the liability reserve of a policy portfolio after the enforced premiums are paid at the valuation date $j = 0$, which can be expressed as

$$L = PV(\text{future benefits}) - PV(\text{future premiums received}) \\ = - \sum_{j=1}^n E[CF(j)] v(j),$$

where n is the maximum remaining policy term for this portfolio. Thus, the reserve L is determined exogenously and equals the

minimal asset value of the insurance company at the valuation date. Frequently, the life insurance authorities in various countries require that the liabilities are valued on a market basis. Therefore, the liabilities of a life insurance portfolio behave more like a portfolio of bonds. In addition, nowadays the life insurance authorities in various countries require that a certain amount of money needs to be set aside as capital at the end of each year. In this paper, we ignore these two issues and assume that the required asset at time 0 is L , consistent with Hurlimann (2002). In turn, this article proposes a feasible asset allocation model to manage L .

Let $I(j)$, $j = 0, \dots, n$, be the accumulation factor from time j to time n , which depends on the asset allocation strategy of the insurance company, and $I(n) = 1$. We define $F(j)$ as the accumulated asset value after adding $CF(j)$ at time j ($j = 1, \dots, n$), and $F(0) = CF(0) = L$. Hence,

$$F(j+1) = F(j) \frac{I(j)}{I(j+1)} + CF(j+1),$$

and the accumulated asset value at time n can be written as

$$F(n) = \sum_{j=0}^n CF(j) I(j).$$

We can describe the first two moments of $F(n)$ with the following lemma.

Lemma 2.1. *Assuming the asset returns and mortality processes are independent, the first two moments of the accumulated asset value at time n are given by*

$$E[F(n)] = \sum_{j=0}^n E[CF(j)] E[I(j)], \quad \text{and}$$

$$\text{Var}[F(n)] = E[\text{Var}[F(n) | i^*]] + \text{Var}[E[F(n) | i^*]],$$

where

$$E[\text{Var}[F(n) | i^*]] = \sum_{j=0}^n \sum_{k=0}^n E[I(j) I(k)] \text{Cov}[CF(j), CF(k)],$$

$$\text{Var}[E[F(n) | i^*]] = \sum_{j=0}^n \sum_{k=0}^n E[CF(j)] E[CF(k)] \text{Cov}[I(j), I(k)],$$

and i^* represents the information set for the asset returns until time n .

Thus, to obtain the closed form of the first two moments of $F(n)$, we first must create the asset return and cash flow models, respectively. The first two moments of the accumulated function, $I(j)$, and the first two moments of the cash flow function, $CF(j)$, then can be calculated. We do not focus on the mortality model herein; we provide the cash flow model and calculations for the corresponding first two moments in Appendix A. Instead, we focus on calculating the accumulate function, for which we require an asset return model, and formulate an investment strategy to produce the accumulated value.

In the asset return model, we adopt the discrete model proposed by Huang and Cairns (2006), which includes three assets: a one-year bond (cash), a long-dated bond, and an equity asset. The log-return rates between time $t - 1$ and t of these assets can be denoted $y(t - 1)$, $\delta_b(t)$, and $\delta_e(t)$, respectively, with the following underlying processes¹:

$$\begin{cases} y(t-1) = y + \phi(y(t-2) - y) + \sigma_y Z_y(t-1) \\ \delta_b(t) = y(t-1) + \Delta_b(t) \\ \quad = y(t-1) + \Delta_b + \sigma_{by} Z_y(t) + \sigma_b Z_b(t) \\ \delta_e(t) = y(t-1) + \Delta_e(t) \\ \quad = y(t-1) + \Delta_e + \sigma_{ey} Z_y(t) + \sigma_{eb} Z_b(t) + \sigma_e Z_e(t), \end{cases}$$

¹ For a general form of this asset model, please refer to chapter 2.1 of Campbell and Viceira (2002).

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