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On parallel asset-liability management in life insurance: a forward risk-neutral approach

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A B S T R A C T

In this paper we discuss the development of a valuation system of asset-liability management of portfolios of life insurance policies on advanced architectures. According to the new rules of the Solvency II project, numerical simulations must provide reliable estimates of the relevant quantities involved in the contracts; therefore, valuation processes have to rely on accurate algorithms able to provide solutions in a suitable turnaround time. Our target is to develop an effective valuation software. At this aim we first introduce a change of numéraire in the stochastic processes for risks sources, thus providing estimates under the forward risk-neutral measure that result in a gain in accuracy. We then parallelize the Monte Carlo method to speed-up the simulation process.

1. Introduction

This work focuses on the development of parallel algorithms for the evaluation of *profit-sharing life insurance policies* (PS policies). This research activity is mainly motivated by the Solvency II project [12], the European project involving the outstanding Control Authorities, which aims to establish a revised set of capital requirements and risk management standards for insurance companies. The new rules of the Solvency II Directive Proposal are increasing more and more the request of stochastic Asset-Liability Management (ALM) models. The ALM, in the Professional Actuarial Specialty Guide [29], is "the practice of managing a business so that decisions on assets and liabilities are coordinated; it can be defined as the ongoing process of formulating, implementing, monitoring and revising strategies related to assets and liabilities in an attempt to achieve financial objectives for a given set of risk tolerances and constraints". This can be obtained by stochastic modelling and simulation.

In this context, we investigate the computational issues in the ALM of PS policies. In these contracts, the benefits which are credited to the policyholder are indexed to the annual return of an investment portfolio: the company invests the reserve in a fund, called the *segregated fund*, and shares the yearly return with the policyholder. A profit-sharing policy is then a derivative contract, with underlying the segregated fund. In Italian insurance market, the crediting mechanism typically guarantees a minimum to the policyholder. It is worth emphasizing that PS policies have been widely analysed in the Solvency II report, since the minimum guarantee feature results in a risk mitigation which allows insurers to reduce the sum needed at the beginning of each year in order to meet the future liabilities. Profit-sharing policies require mark-to-market valuations in order to properly compute all the quantities related to risk management, thus obtaining reliable estimates. The literature on this topic is very rich, we recall [1–3,16,17,20,22] among the others. In particular, we refer to [16,17]. The

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numerical simulation of these financial instruments leads to large-scale computational problems. Our target is to develop a valuation system capable of being properly scaled in order to balance accuracy and efficiency.

The typical computational kernels in the application we consider are Stochastic Differential Equations (SDE) and multidimensional integrals. In [13] we investigated the use of different methods for the numerical solution of the mentioned kernels. Starting from the analysis we carried out in [14] we focused on the development of a parallel algorithm for the evaluation of participating life insurance policies in distributed environments.

In the present paper, we deal with the numerical simulation of a real ALM portfolio; in this framework, we analyse a change of *numéraire* in the stochastic processes for risk sources, since the flexibility of this approach can be particularly valuable in a model with stochastic interest rates. In particular, we analyse the use of the numéraire which defines the *forward risk-neutral measure* [6,18,19,21]. Pricing under the forward measure can provide considerable gains in accuracy, since it allows to discount at a deterministic price deflator, even though the short rate is stochastic [18]. Moreover, we use parallel computing environments to obtain efficient simulation processes.

We propose parallel algorithms for asset-liability management of PS policies portfolios, under both risk-neutral and forward risk-neutral measure, based on the parallelization of Monte Carlo method.

This paper is organized as follows. In Section 2 we outline the asset-liability framework for the evaluation of PS policies; in Section 3 we introduce the stochastic processes for the risk sources; in particular, in Section 3.1 we present the risk-neutral setting, while in Section 3.2 we discuss the change of numéraire, describing the mathematical framework under the forward risk-neutral measure. In Section 4 we discuss the parallel Monte Carlo algorithm. In Section 5 we report the numerical results of a valuation of a real portfolio, in terms of accuracy and efficiency. We test both sequential algorithms based on risk-neutral and forward measure respectively, and the parallel ones implemented on a blade server with twelve processors. Finally, in Section 6 we give some conclusions.

2. Valuation framework

In this section we describe the main features of the mathematical formalization of the Italian contractual standard for profit-sharing policies. Here we briefly report the fundamental elements which are necessary to our discussion. We address to [16,17] for a complete description on the matter.

In a typical asset-liability framework, the basic elements for the evaluation of a PS policies portfolio are:

- the evaluation date t;
- the payment dates $\mathbf{t} := \{t_1, t_2, \ldots, t_m\}$;
- the stream of premiums $\mathbf{X} := \{X_1, X_2, \ldots, X_m\}$;
- the stream of benefits $\mathbf{Y} := \{Y_1, Y_2, \ldots, Y_m\}$;
- the cash-flow stream generated by the segregated fund $\mathbf{Z} := \{Z_1, Z_2, \ldots, Z_m\}$.

From the insurance company point of view, vectors $\mathbf{X}$ and $\mathbf{Z}$ are on the asset side, while $\mathbf{Y}$ is on the liability one. The core of the evaluation problem is the computation of the assets and the liabilities at time t, for the control of the financial equilibrium between them. Then, at time t, the value of the assets:

$$V(t;\mathbf{Z}) + V(t,\mathbf{X})$$

and the value of the liabilities:

$$V(t;\mathbf{Y})$$

must be computed. For providing estimates which are reliable and market-consistent, a mark-to-market stochastic model has to be considered. In such a framework, the value:

$$V_t := V(t;\mathbf{Y}) - V(t,\mathbf{X})$$

that is, the value in t of the difference between the obligations of the company and the obligations of the policyholders, gives the *market value* in t of the outstanding net liabilities of the company: then, it actually represents the amount required to the company at time t in order to meet the future liabilities. For this reason this quantity, called *stochastic reserve*, plays a crucial role for the insurance company.

In order to describe a simplified evaluation framework, we consider a single premium pure endowment insurance contract. Let the policy be written in $t_0 = 0$ for a life of age x; we denote with T the term in years of the contract and with F_t the market value at time t of the segregated fund. Its rate of return in the period $[t-1, t]$ is the random variable

$$I_t = \frac{F_t}{F_{t-1}} - 1$$

According to a typical interest crediting mechanism, the benefits are readjusted at the end of the year t according to:

$$C_t = C_{t-1}(1 + \rho_t), \quad t = 1, \ldots, T \tag{1}$$

where ρ_t is the readjustment rate defined as: