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Quantile hedging for equity-linked life insurance contracts in a stochastic interest rate economy

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ABSTRACT

In this paper we examine equity-linked life insurance contracts in a stochastic interest rate economy via quantile hedging whose purpose is to look for the optimal probability of a successful hedge under initial budget constraint. Most of the existing studies have focused on valuing equity-linked life insurance contracts by quantile hedging or in a framework of stochastic interest rates. However, a few have taken into account simultaneously the two techniques, which make valuing equity-linked life insurance contracts more difficult. We model the term structure of interest rates by classical HJM model that imbeds stochastic interest rate economy into one containing an arbitrary number of additional risky assets. By means of the change of measure approach, we give explicit formulas for the fair values of the following four products: deterministic payoff contract, pure equity-linked life contract, equity-linked life contract with guarantee, equity-linked life contract with minimum guarantees and capped benefits. We find that the explicit formulas are mainly composed of normal distribution functions and two-dimensional normal distribution functions. We also investigate sensibility of the survival probability using data of interest rates, stock prices and life table from China.

1. Introduction

Equity-linked life insurance contracts are now very popular products in North America and the UK and other countries. The benefit of an equity-linked life insurance policy payable at expiration depends upon the market value of a specified equity index or a reference portfolio at maturity. The policyholder does not have to pay taxes on earnings from the portfolio until it is cashed in, usually through death. Cost for policy insurance is a function of the age(s) of the insured, the amount of life insurance coverage wanted, and the payment period wanted. These types of life insurance policies have more flexible terms than whole life insurance. Typically, payments can be made at anytime and in any varying amount. If the equity index or the reference portfolio does well, the payment increases accordingly; if it performs poorly, the payment decreases.

There are a variety of equity-linked insurance products. In this paper, we focus on four forms of typical equity-linked insurance. The first is called by deterministic payoff contract where the payoff of this contract is the same as the traditional life insurance contract and is given by a constant. The second is pure equity-linked policy. This product is in reality not an insurance policy at all, but an investment program in which the insurance company invests the premium in a portfolio, and at expiration pays the policyholder the market value of the investment portfolio (Brennan and Schwartz, 1976). The third, which is the most popular products investigated by many authors, is equity-linked life contract with guarantee. This is an equity-linked life insurance contract whereby the insurer pays upon expiry the current value of the underlying account if the policyholder survives the term with the benefit of a guaranteed return rate on the notional amount, should the market turn downwards. If an equity-linked product has an upper limit on the payoff under the contract, we get the fourth product that is called by equity-linked life contract with minimum guarantees and capped benefits. This product clearly appeals to some issuers since from the issuer's perspective, the capped amount limits the liability under the contract.

The subject of valuing equity-linked life insurance has attracted a lot of attention in the insurance and finance literature. Many attempts have been made to provide consistent valuing approaches for equity-linked life insurance using financial and economical methods. Brennan and Schwartz (1976) and Boyle and Schwartz (1977) are the first papers that investigate the problem of premium calculations for the equity-linked insurance contracts. By decomposing benefit of those products into a sure amount and an immediately exercisable call option on the reference portfolio, traditional Black-Scholes model is applied to evaluate life insurance products embedded with some financial guarantees. One of the most used underlying assumptions is

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the completeness of the market, which means that the insurer is "risk-neutral" with respect to the mortality risk.

However, many studies have questioned this assumption. The payoff of equity-linked life insurance depends on both future prices of the underlying financial assets and the policyholder's mortality, retirement, survival to a certain date, etc. Many studies have pointed out that imperfect hedging approach should be applied to price equity-linked products since the mortality risk makes the market incomplete. More and more authors have developed different pricing and hedging strategies in incomplete markets. Møller (1998) employs the mean-variance hedging method to evaluate equity-linked life insurances. More recently, Gaillardetz and Lin (2006) present a market consistent valuation technique for equity-linked products, while Young and Zariphopoulou (2002) and Moore (2009) evaluate these products based on the principle of equivalent utility theory.

Quantile hedging technique of Follmer and Leukert (1999) is another pricing approach that is perfectly suited to pricing equity-linked insurance contracts. In most of equity-linked life insurance products setting, insurers are subjected to a blend of mortality risk and market risk and the writer of the contract has to optimize hedging strategies given a limited initial capital. Quantile hedging can combine those two sources of risk. The main characteristic of quantile hedging is that it can get the optimal probability of a successful hedge under a given initial capital, or to fix some acceptable level of financial risk and then choose the clients for the contracts accordingly to preserve this level of risk. There are a number of papers adapting the quantile hedging approach to the insurance setting. Among them, Alexander Melnikov has in a sequence of papers devoted much to applying quantile hedging to valuing equity-linked life insurance contracts. Melnikov (2004) and Kirch and Melnikov (2005) devote themselves to the problem of hedging contingent claims in the framework of a two factors jump-diffusion model under initial budget constraint. Melnikov and Romaniuk (2006) apply quantile hedging techniques to price a unit-linked contract with payoff conditioned on the client's survival to the contract's maturity. Melnikov and Romanyuk (2008) use the efficient hedging methodology for optimal pricing and hedging of equity-linked life insurance contracts whose payoff depends on the performance of several risky assets. Wang (2009) also uses quantile hedging for guaranteed minimum death benefits under various assumptions.

In this paper we take up the issue of pricing equity-linked insurance contracts via quantile hedging under stochastic interest rates. Bacinello and Ortu (1993) and Nielsen and Sandmann (1995) also investigate equity-linked life insurance through a model with stochastic interest rates. Additionally, Lin and Tan (2003) consider pricing of equity-indexed life products, in which the external equity index and the interest rates are driven by stochastic differential equations. Mao and Ostaszewski (2008) propose a model that integrates supply and demand considerations, stochastic interest rates, stochastic investment return, and mortality rates, to examine the pricing of equity-linked life insurance in a partial equilibrium framework. The term structure of interest rates is based on the Cox et al. (1985) model. However, in a stochastic interest rate framework, the issue of valuing equity-linked life insurance via quantile hedging has not been studied very much.

Our set-up is the same as in Bacinello and Persson (2002) which is standard in the theories of financial economics. We use the term structure of interest rates proposed by Heath et al. (1992) (HJM, hereafter), where an asset and a default free bond are traded and prices of the bonds and the interest rates can be deduced. The sources of uncertainty come from the interest rate and asset to which the policy is linked. According to the HJM-model, the development of the forward rates and the initial term structure are given. By assuming that no arbitrage opportunities exist, the drift term of the forward rate is restricted in a way so that valuing can be done from knowledge of the volatility processes of the forward rate and the initial term structure. The combination of stochastic interest and quantile hedging method makes valuing equity-linked life insurance contracts more complex than the model with constant interest rates. The main characteristic of our method is that we adopt the method of change of probability measures to get fair values of the products considered. The fair values of these policies are calculated as expectations under a new probability measure. We find that it is easy to obtain a close-form solution. We also present some comparative static results in empirical analysis.

The remainder of the paper is organized as follows. The next section presents notation, reviews the classical HJM approach and formulates the general quantile hedging technique. Section 3 contains our main theoretical results. We present four explicit formulas of the fair values of the products considered. In Section 4, the characteristic of the suggested results is evaluated by considering a range of realistic data from China. Section 5 concludes and discusses other potential practical applications of the paper's results.

2. Market model and hedging strategies

Firstly, we give some definitions and results concerning interest rates and specify the term structure of interest rates. Then we present insurance setting and review quantile hedging technique that we will use hereafter.

2.1. Financial setting

In this subsection we introduce, first of all, the basic assumptions concerning the financial set-up. We take the HJM setup as its point of departure. As in Persson (1995) and Bacinello and Persson (2002), we assume that the insurer maintains the investment of the asset base in a well-diversified and well-specified reference portfolio that is composed of two types of assets, including one bond and equities. We fix a complete probability space$(\Omega, \mathbb{F}, \mathcal{P})$, where $\mathcal{P}$ is the real-world probability measure. Let $\mathcal{T}$ denote the trading interval [0, T] of the model. Let $\{B_1(t)\}_{t\in\mathcal{T}}$ denote a standard Brownian motion defined on $(\Omega, \mathbb{F}, \mathcal{P})$ with respect to the $\mathcal{P}$-augmentation of its natural filtration $\mathbb{F} := \{\mathbb{F}_t\}_{t\in\mathcal{T}}$.

Suppose the relationship between forward rates and market prices of unit discount bonds is

$$M(t,\tau) = \exp\left[-\int_t^\tau f(t,v)dv\right] \tag{1}$$

Given an initial forward rate curve $f(0,\tau)$, the family of continuously compounded forward rates $f(t,\tau)$,$0<t<\tau<T$, is given by the following stochastic integral equation

$$f(t,\tau) = f(0,\tau) + \int_0^t \mu^f(u,\tau)du + \int_0^t \sigma^f(u,\tau)dB_1^P(u) \tag{2}$$

The drift and volatility processes, $\mu^f(t,\tau)$ and $\sigma^f(t,\tau)$, respectively, are assumed to be adapted with respect to $\mathbb{F}_t$, and jointly measurable and uniformly bounded. The spot interest rate at time t, $\{r(t)\}_{t\in\mathcal{T}}$, is given by the instantaneous forward rate of a forward contract, i.e. $r(t) = f(t,t)$. The dynamics of the price process $\{R(t)\}_{t\in\mathcal{T}}$ for a zero-coupon bond is described by

$$R(t) = \exp\left[\int_0^t r(u)du\right] \tag{3}$$

When considering the return process of an asset traded in a financial market, we assume that the new probability space is $(\Omega, \mathbb{G}, \mathcal{P})$, where $\mathbb{G} := \{\mathbb{G}_t\}_{t\in\mathcal{T}}$ is the augmented filtration that is generated by a two dimensional Brownian motion $\{B_1^P(t), B_2^P(t)\}_{t\in\mathcal{T}}$ initialized at zero. The dynamics of the underlying market price process of the reference asset backing the participating contract