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Improved linear regression method for estimating Weibull parameters

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ABSTRACT

In the linear regression method for estimating parameters of a Weibull distribution, multiple flaw distributions may be further evidenced by derivation form the linearity of data from a single Weibull distribution. In this paper, a new technique of estimating multiple Weibull parameters is conducted, compared to commonly used regression probability estimator.

1. Introduction

Weibull statistics has become a well-established characterization tool in the field of fracture strength of ceramics. Experiences have shown that mechanical and fracture properties of brittle materials are strongly affected by the component size and the Weibull weakest link model properly considers the size effect of the components [6]. Based on physical assumptions [7], the Weibull equation describes the relationship between the probabilities of failure P_f of a brittle material under uniaxial tensile stress of σ. It thus predicts the inherent dispersion in fracture strength of brittle materials. The simplified two parameters Weibull equation

$$P_f \equiv F(\sigma) = 1 - \exp\left[-\left(\frac{\sigma}{\sigma_0}\right)^m\right], \tag{1}$$

has been widely used in estimating failure chance of ceramic components. The two parameters in Eq. (1), the so called Weibull parameters, determine the shape and location of the cumulative distribution function $F(\sigma)$. The Weibull modulus m, sometimes called the shape parameter, has a value between 5 and 20 for technical ceramics [4]. On a normalized scale, a higher m leads to a steeper function and thus a lower dispersion of fracture stresses. The scale parameter σ_0 is closely related to the mean fracture stress, influences the variance of the fracture stress, i.e. the steepness of the function: a smaller σ_0 means – on an absolute scale – a lower dispersion.

Once a set of N experimentally measured fracture stresses are obtained, it is desirable to fit the Weibull equation (Eq. (1)) to these observations, i.e. to determine the two parameters m and σ_0, knowledge of which leads to a complete characterization of the material for the given volume.

There are several methods available in the literature for the determination of these two parameters [2,3]. The most widely used is the linear regression method due to its simplicity. The linear regression method is based on the fact that Eq. (1) can be written as a linear form when take the logarithm twice:

$$\ln\left[\ln\left(\frac{1}{1-P_f}\right)\right] = m\ln\sigma - m\ln\sigma_0. \tag{2}$$

So, the measured fracture stresses are ranked in ascending order, and a probability of failure P_i is assigned to each stress σ_i. The Weibull modulus can be obtained directly from the slope term in Eq. (2), and the scale parameter can be deduced from the intercept term.

Several prescribed probability estimators has been used as the P_i-value in different literatures. The following four expressions are often applied to define the probability estimator [2,3]:

$$\begin{aligned} P_i &= \frac{i-0.5}{n}, \\ P_i &= \frac{i}{n+1}, \\ P_i &= \frac{i-0.3}{n+0.4}, \\ P_i &= \frac{i-0.375}{n+0.25}, \end{aligned} \tag{3}$$

where P_i is the probability of failure for the ith-ranked stress datum. The relative merits of these estimators have been investigated by several authors with actual or computer-generated strength data.

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It has been shown that the first of Eq. (3) gives the least bias and is therefore preferred.

However, advanced ceramics typically contain two or more active flaw distributions each with an independent set of parameters [1]. The objective of this paper is to model these conditions and estimate the multiple flaw distributions using linear regression method.

2. Linear regression method

The data applied in this paper is extracted from Mirhabibi and Rand latest work on fracture strength analysis [5]. Fig. 1 gives the Weibull probability plot using the first of Eq. (3) for the data. Obviously, the fracture strengths are not actually near a linear line. The regression line has a coefficient of determination (R^2) equal to 0.934. That is, 93.4% of the variability in the data points is described by the drown regression line.

The estimated parameters $\hat{m}$ and $\hat{\sigma}_0$ using regular linear regression method are 6.82 and 59.78, respectively. What if one fit a curve to this data?

Fig. 2 shows two curves, a quadratic and a cubic one, fitted to the data. Results show that the R^2 doesn't change seriously in the quadratic curve, $R^2 = 0.935$, while in the cubic curve it increases to 0.956. This means that the strength data are better described with a cubic curve than a line.

Let X and Y be two independent random variables. X has a Weibull distribution with parameters m and σ_0 and Y is distributed Weibull with parameters n and θ_0, that is

$$P_s^X \equiv 1 - F(\sigma) = \exp\left[-\left(\frac{\sigma}{\sigma_0}\right)^m\right],$$

$$P_s^Y \equiv 1 - F(\theta) = \exp\left[-\left(\frac{\theta}{\theta_0}\right)^n\right].$$

If $X_1, X_2, \ldots, X_{N_1}$ and $Y_1, Y_2, \ldots, Y_{N_2}$ be two random samples from X and Y respectively, then

$$P_s^{X,Y} = P_s^X.P_s^Y \equiv \exp\left[-\left(\frac{\sigma}{\sigma_0}\right)^m - \left(\frac{\theta}{\theta_0}\right)^n\right]. \quad (4)$$

Now, if an experimenter doesn't have any knowledge about the way data are provided, he would order the whole $N = N_1 + N_2$ data and try to fit one regression line in order to estimate Weibull parameters using the regression method described in the introduction. But, Eq. (4) results

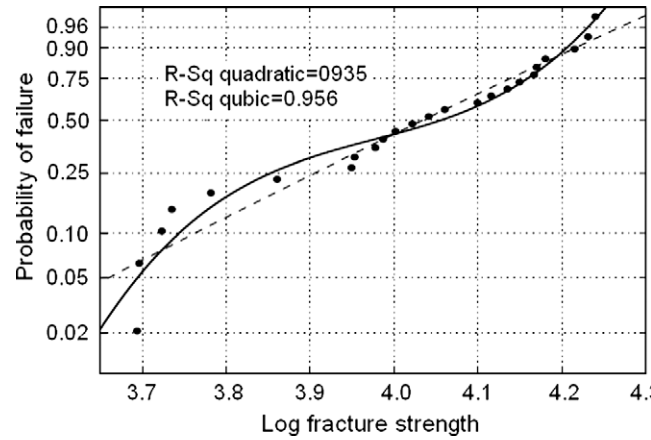

Fig. 2. Curved regression lines fitted to the strength data.

$$\ln\left[\ln\left(\frac{1}{1 - P_{f,N}^{X,Y}}\right)\right] = m\ln\sigma - m\ln\sigma_0 + n\ln\theta - n\ln\theta_0. \quad (5)$$

This is exactly what is needed in order to fit a curve to the strength data.

3. Weibull parameters estimation

It's a common mistake when an experimenter unintentionally affects the strength data. He may use two or more strength measure machines, or two or more employees to provide the ceramics, or even produces the ceramics in two different days. All of these, along with applying different sizes, will affect the produced ceramics and can cause multiple flaw distributions since the condition of producing ceramics has not been controlled. If he was lucky, he can recognize the deviation form the linearity of the data from a single Weibull distribution. But if the deviation was small, he uses a regular Weibull model and causes a biased estimation.

With the method provided in former section it's easy to find the Weibull parameters estimation using linear regression method. Simply fit three linear lines to the data in a way that the lines be the best lines describe a curve fit. Fig. 3 shows the three linear regression lines fitted to the former data and Table 1 gives the Weibull parameters estimates.

In subclass 1 of the data which is consists of 4 measured ceramics, the fitted line has a slope equals to 13.28. It's also the estimation of m, however the small number of specimens in this group

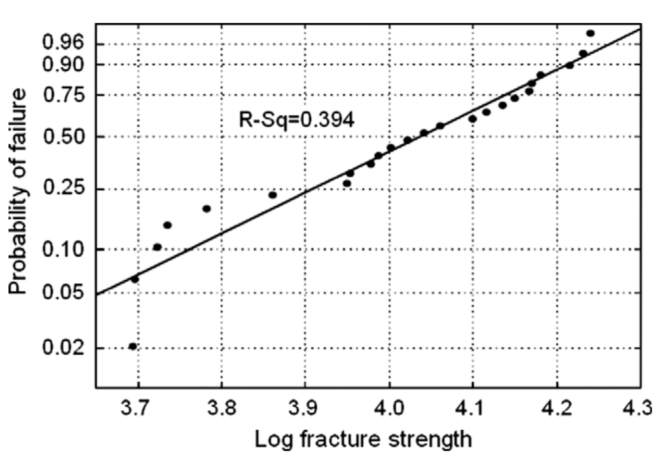

Fig. 1. Linear regression line fitted to the strength data.

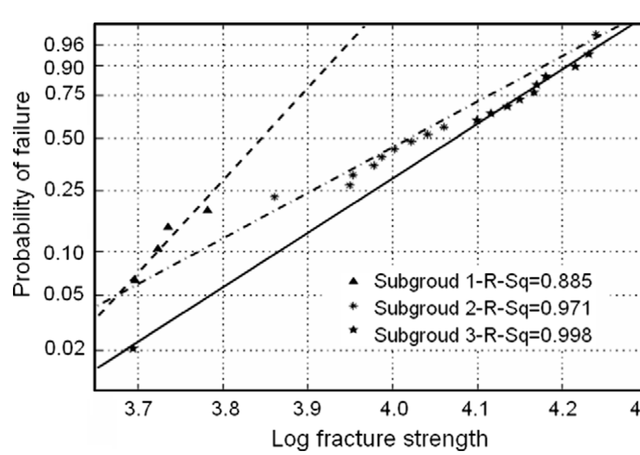

Fig. 3. Two linear regression lines fitted to the strength data.