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A constrained recursive pseudo-linear regression scheme for on-line parameter estimation in adaptive control

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ABSTRACT

In adaptive control of systems with poles close to the unit circle, application of the recursive estimation techniques can lead to excursions of the poles of the identified model outside the unit circle even when the process is open loop stable. These excursions can be of two types. The poles of the deterministic component of the model can drift outside unit circle even when the process has no unstable modes. Alternatively, the poles and/or zeros of the unmeasured disturbance (noise) model can drift outside the unit circle. In either case, the identified model is not suitable for on-line controller adaptation. In this work, a novel constrained recursive formulation is proposed for on-line parameter estimation based on the pseudo-linear regression (PLR) approach. The efficacy of the proposed approach is demonstrated by conducting experimental studies on a benchmark laboratory scale heater-mixer setup. The analysis of the open and closed loop experimental results reveals that the proposed constrained parameter estimation scheme provides a systematic and computationally attractive approach to ensure that the identified model parameters are restricted to the feasible region.

1. Introduction

The field of adaptive control with on-line recursive parameter estimation has received considerable attention in the past three decades [1,2]. The ease of implementation of on-line parameter estimation algorithms developed over the years has made adaptive predictive control a competent alternative to nonlinear process control [3–6]. Parameter estimation is of paramount importance in the context of an adaptive control system and therefore the need to generate consistent parameter estimates cannot be ignored. The recursive least square (RLS) estimation of *linear in parameter* models, such as ARX or FIR, and its extensions to *nonlinear in parameter* models (pseudo-linear regression or PLR) have been extensively employed for this purpose [2] and the convergence results of such schemes have been discussed by Ljung [7]. These methods belong to the class of *recursive prediction error methods*, which fail to generate meaningful estimates when the predictors are unstable.

In the certainty equivalence adaptive control approach, the controller is designed under the assumption that the identified linear model gives an exact representation of the process dynamics in the neighborhood of the current operating point [8]. In the review of adaptive control presented at CPC-V, Ydstie [8] identifies the *admissibility problem* as one of the key issues that must be addressed in certainty equivalence control to achieve bounded input bounded output (BIBO) stability and robustness with respect to unmodelled dynamics and disturbances. This implies that the estimated model must be well behaved (controllable or stabilizable). Further, Ydstie [8] identifies the *instability of the parameter estimator* or the *parameter drift* as another important issue that must be addressed while developing an adaptive control scheme. Ydstie [8] states that the parameter drift can lead to a situation where parameters converge to infeasible or unstable solutions.

The on-line parameter drifts can be traced to factors such as insufficient input excitation and variance errors in the parameter estimates. In on-line recursive parameter estimation, the concept of forgetting factor is often used to adjust the length of past data influencing the model parameters and, in turn, the rate of on-line model adaptation. A usual dilemma associated with the choice of the forgetting factor is selecting between better closed loop performance with larger variance errors in the model parameters and sluggish closed loop performance with relatively well behaved estimator. When the system under consideration shows significant time varying characteristics, faster adaptation (or shorter memory length) can potentially generate a tighter control of the process. Thus, faster adaptation is desirable from the closed loop performance viewpoint. However, smaller the model memory, larger are the variance errors associated with the estimated model parameters. In the context of adaptive control of systems with poles close to the unit circle, relatively smaller values of forgetting

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factor can lead to excursions of poles/zeros of the estimated model outside the unit circle due to relatively large variance errors. These excursions can be of two types:

- Model with respect to manipulated inputs: The poles of the deterministic component of the model can drift outside unit circle even when the process transfer function has no unstable poles. In this case the identified model is not suitable for controller design.
- Model for unmeasured disturbances: Poles and/or zeros of the unmeasured disturbance (noise) model can drift outside the unit circle. The unmeasured disturbances are often modelled as stationary stochastic processes and, by spectral factorization theorem, the unmeasured disturbance models are required to be stable and inversely stable [7]. Any violation of this condition can lead to difficulties in predicting the effect of past disturbances on the future process behavior.

Thus, these excursions can have detrimental effects on the resulting closed loop behavior. These difficulties are often ignored while developing the parameter estimator and handled later at the implementation stage by employing some ad hoc fixes.

The problem of parameter drift can be avoided either (i) by adding a deliberate perturbation to the setpoint or manipulated inputs, which induces continuous variability in the controlled outputs, (ii) use of fixed pole models or (iii) by using parameter projection to ensure that parameters do not go outside a bounded region. The use of a Laguerre filter based model has also been advocated to deal with this problem as these models are stable and robustly stabilizable provided the unmodelled dynamics are stable [8]. One drawback of Laguerre or any other orthonormal basis filter based model is that these models have fixed poles [10]. As a consequence, the parameter adaptation is restricted to adaptation of system zeros and steady state gains. When a process is operated over a wide operating range, time constants can change significantly and adaptive control based on fixed pole models may not provide uniformly satisfactory performance over the entire operating range. Ydstie [8] has proposed a systematic approach to deal with the constraints on parameters by posing the parameter estimation problem as a constrained optimization in the moving horizon framework. However, the main difficulty with this formulation is that it results in a highly nonlinear constrained optimization problem and solving it on-line in real-time may not be feasible when the sampling time is relatively small. For the model building task to keep up with the information flow, the on-line computations must be done is such a way that the processing of measurements from one sample is completed within a significantly small fraction of a sampling interval [7]. This calls for a computationally efficient recursive parameter estimation scheme that can handle constraints on the estimated parameters. Moreover, estimation of the appropriate *arrival cost*, which connects the previous parameter estimates with the new problem, is an open issue.

The parameter estimation scheme based on recursive least squares can be regarded as a form of the Kalman filter [2]. It is a well acknowledged fact that the Kalman filter or its extensions, such as extended Kalman filter, do not handle bounds or constraints that may have to be imposed on state or parameter estimates. The nonlinear dynamic data reconciliation (NDDR) [11] and moving-horizon estimation (MHE) [12] techniques were introduced to include the knowledge of inequality and equality constraints on states, parameters and disturbances in the state estimation problem. These approaches pose the state and parameter estimation problem as a nonlinear constrained optimization problem over a moving data window in the past. Recently, Vachhani et al. [13] have proposed a recursive NDDR technique, which combines the computational benefits of the extended Kalman filter and the constraint handling properties of NDDR. The constrained optimization problem is formulated over only one sampling instant thereby achieving significant reduction in on-line computation time. Thus, in the context of on-line constrained parameter estimation for adaptive control, the RNDDR formulation appears to be a promising approach.

In this work, we propose a constrained recursive pseudo-linear regression (C-RPLR) formulation that not only ensures the stability of the resulting models but also is computationally efficient [14]. The Jury's stability criterion is used to formulate the constraints in the C-RPLR scheme. When the model order is less than or equal to two, the resulting constrained optimization problem can be formulated as a QP problem. Towards employing the QP formulation for models of order greater than two, we propose a sequential estimation scheme, which rearrange the computations in such a way that QP formulation can be applied at each step. The proposed C-RPLR approach ensures that

- When the process under consideration is locally open loop stable, the parameter drift does not cause excursion of model poles to the unstable region.
- When the unmeasured disturbances are modelled as a transfer function driven by a white noise process, the transfer function remains stable and inversely stable as necessitated by the *spectral factorization theorem*.

To begin with, we demonstrate the effectiveness of the proposed C-RPLR scheme on a batch of data obtained from a benchmark heater-mixer setup [15]. Since the objective of the proposed constrained recursive estimation strategy is to ensure good performance of an adaptive control scheme, we proceed to demonstrate the performance of an adaptive model predictive controller that employs C-RPLR scheme for parameter estimation.

The paper is organized as follows. We begin by presenting the preliminaries of prediction error method (PEM) and RPLR based parameter estimation schemes in Section 2. In Section 3, we describe the proposed constrained recursive pseudo-linear regression (C-RPLR) strategy. The application of C-RPLR to identification of models with OE and ARMAX structure is described in Section 4. The results of experimental studies have been presented in Section 5 and the main conclusions reached through analysis of the results are presented in Section 6.

2. Brief review of recursive parameter estimation

In this work, we propose to model an open loop stable MIMO process as multiple MISO models. Thus, to keep the notations simple, we consider an open loop stable MISO system that is modelled using a linear time series model of the form

$$y(t) = \sum_{i=1}^{m} G_i(q^{-1}, \boldsymbol{\theta}^{(i)})\mathbf{u}_i(t) + H(q^{-1}, \boldsymbol{\theta}_h)e(t) \tag{1}$$

where $y(t) \in R$ represents measured output, $\mathbf{u} \in R^m$ represents manipulated or known inputs while $\mathbf{u}_i$ represents the ith input in the vector $\mathbf{u}$, $G_i(q)$: $i = 1,2,\ldots,m$ represent stable transfer functions in unit delay operator q^{-1}, $\boldsymbol{\theta}^{(i)} \in R^{n_i}$ represents parameter vector of $G_i(\cdot)$, $H(q,\boldsymbol{\theta}_h)$ represents noise transfer function with parameter vector $\boldsymbol{\theta}_h \in R^{n_h}$ and $\{e(t)\}$ represents a zero mean Gaussian white noise sequence with variance σ^2. The above model implicitly assumes that signal $\{\upsilon(t)\}$ defined as

$$\upsilon(t) = H(q^{-1}, \boldsymbol{\theta}_h)e(t)$$

is a stationary process and $H(q^{-1},\boldsymbol{\theta}_h)$ is a stable and inversely stable transfer function [7]. The model based control techniques require that the parameter vector, $\boldsymbol{\theta}$, represented as