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Cournot games with linear regression expectations in oligopolistic markets

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Abstract

In this paper, a Cournot game in an oligopolistic market with incomplete information is considered. The market consists of some producers that compete for getting higher payoffs. For optimal decision making, each player needs to estimate its rivals' behaviors. This estimation is carried out using linear regression and recursive weighted least-squares method. As the information of each player about its rivals increases during the game, its estimation of their reaction functions becomes more accurate. Here, it is shown that by choosing appropriate regressors for estimating the strategies of other players at each time-step of the market and using them for making the next step decision, the game will converge to its Nash equilibrium point. The simulation results for an oligopolistic market show the effectiveness of the proposed method.

Keywords: Game theory; Cournot game; Incomplete information; Oligopoly market; Estimation; Nash equilibrium

1. Introduction

The first dynamic model in the field of oligopoly games was introduced by Cournot [17]. In this game, the players make their decisions according to the strategies of their rivals using naive expectations model where it is assumed that the rivals repeat their last strategy in the next step of the game.

Complicated dynamics of duopoly games were studied for different demand functions [10,11,21,25]. The dynamic of a three-player game was discussed in Ref. [26]. In Refs. [1,2,7], oligopoly dynamic models were considered with the simple assumption of naive expectations. In Ref. [1], the dynamics of three- and four-player games in a Cournot model were studied assuming linear cost functions for the players. In Ref. [7], the result was extended to an n-player game with linear cost functions. The basins of attraction for the multiple Nash equilibrium points were discussed in Ref. [2] where the players use naive expectations and do not estimate the rivals' behaviors. In Ref. [12], a duopoly game with adaptive adjustment of the rival's behavior was taken into account. A review of the studies on complicated dynamics of oligopoly games can be found in Ref. [27,28]. Many works have considered the case of homogeneous

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expectations of the rivals, where the players use the same rule for behavior adjustment [3–5,21,27]. This approach is not useful for the case that the players have different decision making strategies.

In a more complex model of dynamic game, it is assumed that none of the players have complete information of the game, and thus, they should act adaptively. In an adjustment process based on bounded rationality, the players estimate their marginal profit to amend their performance [8,10].

In Ref. [13], an oligopoly game with incomplete information for the sellers was studied. The players use naive expectations to predict the behavior of other players. A new method, called local monopolistic approximation, is proposed for profit maximization. The dynamics of a triopoly game with heterogeneous players were discussed in Ref. [19]. In that work, three players with different expectations were used, namely bounded rational, adaptive, and naive expectations. The stability of the game and its chaotic behavior were analyzed. However, none of the agents estimate the action of the rivals and they only use the last action of their rivals for decision making.

A nonlinear duopoly game with heterogeneous players was considered in Ref. [35]. One of the players use naive expectations to predict its rival and the other one is a bounded rational player which uses a replicator equation called myopic. The existence and the stability conditions of the equilibrium point were discussed. However, the players do not estimate the next action of the rivals.

In Ref. [9], an oligopoly electricity market was analyzed where the replicator dynamics were used for decision making. However, the players were assumed to have naive expectations for predicting their rival's decisions.

One of the most important issues in a game is the stability analysis of the Nash solution and finding a stable region within which the convergence of the game trajectory to the Nash equilibrium point is guaranteed [5,7,20,33,34]. The stability of the Nash equilibrium in a linear oligopoly model of an n-player game was shown in Ref. [31]. This result was generalized to the stability of equilibrium point in linear oligopolistic models when players use adaptive expectations instead of best reply [29,30].

The prospect of this paper is to propose an estimation model to predict the next step behaviors of the rivals of a player in a Cournot game, and to demonstrate the game convergence to its Nash equilibrium point. The strategy of the players, who are sellers, is the quantity that they offer, and it is assumed that the sellers do not know their rivals' bidding strategies in the next time-step. It is further assumed that the players are rational, meaning that they are after maximizing their own profits at each stage. An intelligent player is designed which predicts the strategies of the rivals for the next stage and chooses the strategy that maximizes its payoff for that stage. Assuming that all the players of the game use the proposed estimation model, the method guarantees the asymptotic convergence to the Nash equilibrium through repeated application of market rounds. In the simulation part, three different cases are considered: (1) three similar market agents that are designed based on the proposed algorithm; (2) three agents, two of them acting based on the proposed method, and one based on naive expectations of the rivals' strategies; and (3) three agents, two of them based on the proposed method, and one based on the gradient dynamics. It is shown that while the Nash equilibrium in each of the three cases is stable, the average and cumulative payoffs of the agent based on the proposed method are better than those with the gradient dynamics and naive expectations agents.

The organization of this paper is as follows: Section 2 provides the problem formulation and preliminaries of the work. In Section 3, strategy estimation based on a linear regression model is presented. In Section 4, decision making based on payoff maximization is discussed. Section 5 demonstrates the proof of the game convergence to the Nash equilibrium. Section 6 includes the simulation results for the three cases studies. Finally, the conclusion of the paper is provided in Section 7.

2. Problem formulation and preliminaries

Consider a market consisting of n sellers (players). The market clearing price can be calculated as follows:

$$\lambda(t) = \lambda_0 - \sum_{i=1}^{n} \alpha_i q_i(t) \tag{1}$$

where q_i is the production quantity and $\alpha_i > 0$ is the market power/share parameter of the ith player. The scalar λ_0 is the base price in the market. Eq. (1) implies that increasing the production quantity results in decreasing the market price. The payoff or profit of the ith player can be calculated as follows:

$$\pi_i(t) = \lambda(t) q_i(t) - C_i(q_i(t)), \quad i = 1, \ldots, n \tag{2}$$