Risk analysis and valuation of life insurance contracts: Combining actuarial and financial approaches

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1. Introduction

Interest rate guarantees are a very common product feature within traditional participating life insurance contracts in many markets. There are two major types of interest rate guarantees:

The simplest interest rate guarantee is a so-called point-to-point guarantee, i.e. a guarantee that is only relevant at maturity of the contract. The other type is called cliquet-style (or year-by-year) guarantee. This means that the policy holders have an account to which every year at least a certain guaranteed rate of return has to be credited.

Cliquet-style guarantees of course may force insurers to provide relatively high guaranteed rates of interest to accounts to which a big portion of the past years’ surplus has already been credited. Adverse capital market scenarios of recent years appeared to have caused significant problems for insurers offering this type of guarantee. Therefore, the analysis of traditional life insurance contracts with cliquet-style guarantees has become a subject of increasing concern for the academic world as well as for practitioners.

There are so-called financial and actuarial approaches to handling financial guarantees within life insurance contracts. The financial approach is concerned with risk-neutral valuation and fair pricing and has been researched by various authors such as Bryis and de Varenne (1997), Grosen and Jørgensen (2000), Grosen and Jørgensen (2002) or Bauer et al. (2006). Note that the concept of risk-neutral valuation is based on the assumption of a perfect (or super-) hedging strategy, which insurance companies normally do not or cannot follow (cf. e.g. Bauer et al. (2006)). If the insurer does not or cannot invest in a portfolio that replicates the liabilities, the company remains at risk and should therefore additionally perform some risk analyses. The actuarial approach focuses on quantifying this risk with suitable risk-measures under an objective ‘real-world’ probability-measure, cf. e.g. Kling et al. (2007a) or Kling et al. (2007b). Such approaches also play an important role e.g. in financial strength ratings or under the new Solvency II approach. Amongst others, Gatzert and Kling (2007) investigate parameter combinations that yield fair contracts and analyze the risk imposed by fair contracts for various insurance contract models, starting with a simple generic point-to-point guarantee and afterwards analyzing more sophisticated Danish- and UK-style contracts. Kling (2007) focuses on traditional German insurance contracts where the interdependence of various parameters concerning the risk exposure of fair contracts is studied. Gatzert (2008) extends the work from Gatzert and Kling (2007) where an approach to ‘risk pricing’ is introduced using the ‘fair value of default’ to determine contracts with the same risk exposure. However, this risk measure neglects real-world scenarios and is only concerned with the (risk-neutral) value of the introduced default put option. Whilst Gatzert (2008) analyzes some real-world risk generated by
the considered contracts, the risk exposure is not incorporated in
the pricing procedure.

Barbarin and Devolder (2005) introduce a methodology that
allows for combining the financial and actuarial approach. They
consider a contract similar to Bryis and de Varenne’s (1997) with
a point-to-point guarantee and terminal surplus participation.
To integrate both approaches, they use a two-step method of
pricing life insurance contracts: First, they determine a guaranteed
interest rate such that certain solvency requirements are satisfied,
using value at risk and expected shortfall risk measures. Second, to
obtain fair contracts, they use risk-neutral valuation and adjust the
participation in terminal surplus accordingly.

The inceptive aim is to provide an analytical interpretation
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using value at risk and expected shortfall risk measures. Second, to
obtain fair contracts, they use risk-neutral valuation and adjust the
participation in terminal surplus accordingly.

We first specify our asset model under the real-world proba-
bility measure $\mathbb{P}$ and then switch to the risk-neutral measure $\mathbb{Q}$
which will be used for valuation purposes. We consider a proba-

ability space $(\Omega, F, \mathbb{P}, \mathbb{F})$ with the natural filtration $\mathbb{F} = \mathbb{F}_t = \sigma(W_s (s), s \leq t)$ generated by independent $\mathbb{P}$-Brownian Motions $W_t$ and $\tilde{W}_t$ and let $r(t)$ denote the short-rate and $\mathbb{S}(t)$ the value of the stock at time $t$.

We further consider a bond portfolio consisting of different
zero-bonds. Hence we need to determine $p(t, T)$, the price at time
$t$ of a zero-bond with maturity $T$. We assume that $p(t, T) =
F(t, r(t))$ holds for some smooth function $F(t, r(t))$. Since the short-rate is not observable on the market we may not be able to hedge derivatives on the short rate (e.g. zero-bonds) by investing in the underlying itself as it could be done e.g. in a Black–Scholes framework. Investing in the bank account instead would result in an incomplete market.

2. Model framework

2.1. Insurance company

Following Kling et al. (2007a), we consider a simplified ‘balance
sheet’ of the insurance company as follows:

<table>
<thead>
<tr>
<th>Assets</th>
<th>Liabilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L(t)$</td>
<td>$A(t)$</td>
</tr>
<tr>
<td>$B(t)$</td>
<td>$A(t)$</td>
</tr>
<tr>
<td>$R(t)$</td>
<td></td>
</tr>
</tbody>
</table>

Here, $A(t)$ denotes the market value of the company’s assets.
$L(t)$ represents the insurer’s liabilities measured by the actuarial
reserve for the insurance contracts. Every year $L(t)$ has to earn
at least a fixed guaranteed interest rate $\lambda$, thus $L(t + 1) \geq
L(t)(1 + \lambda)$. The insured can participate in the insurer’s asset return exceeding the guaranteed rate in two ways: By regular surplus
participation if in any year more than the guaranteed interest rate
$\lambda$ is credited to the account $L$ and by terminal surplus partici-
patation. $B(t)$ models a collective terminal surplus account, which is used to provide additional surplus participation at the maturity of a client’s contract. This account may be reduced at any time in order to ensure the company’s liquidity which leaves $B(t)$ to be an optional
bonus payment and $B(t) \geq 0$ for all $t$. The residual value $R(t) =
A(t) - (L(t) + B(t))$ denotes the (hidden) reserves of the life insurer.

2.2. Financial market

We now introduce the model for the financial market and the
financial instruments in the insurer’s asset portfolio. We allow
investment in the money market, bonds and stocks. We use the
Vasicek (1977) model for stochastic interest rates and a Geometric
Brownian Motion (cf. Black and Scholes, 1973) for a reference stock
or stock index.

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The asset model is then given by the stochastic differential equations (SDEs)

$$
\begin{align*}
 dr(t) &= a(b - r(t)) dt + \sigma dz_t \\
 dS(t) &= S(t)(\mu dt + \sigma dW_t(t)) + \sqrt{1 - \rho^2} d\tilde{W}_t(t)
\end{align*}
$$

with correlation $\rho \in [-1, 1]$. To simplify notation, we let $W_t(t) :=
\rho W_t(t) + (\sqrt{1 - \rho^2}) \tilde{W}_t(t)$. Thus, for $t_1 \leq t_2$, a closed form solution of the above SDEs is given by

$$
r(t_2) = e^{-\alpha(t_2 - t_1)} r(t_1) + b(1 - e^{-\alpha(t_2 - t_1)}) + \sigma \int_{t_1}^{t_2} e^\alpha dW_t(u)
$$

$$
S(t_2) = S(t_1) \left\{ \left(1 - e^{-\alpha(t_2 - t_1)} \right) + \sigma \int_{t_1}^{t_2} e^{\alpha u} dW_u \right\}.
$$

A money market investment is then modeled by an investment in the short rate: $\beta(t) = e^{\int_0^t \langle r(s) \rangle ds}$

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zero-bonds. Hence we need to determine $p(t, T)$, the price at time
$t$ of a zero-bond with maturity $T$. We assume that $p(t, T) =
F(t, r(t))$ holds for some smooth function $F(t, r(t))$. Since the short-rate is not observable on the market we may not be able to hedge derivatives on the short rate (e.g. zero-bonds) by investing in the underlying itself as it could be done e.g. in a Black–Scholes framework. Investing in the bank account instead would result in an incomplete market.

By constructing a portfolio with no instantaneous risk (e.g. con-
sisting of two zero-bonds with different maturities) and applying
no arbitrage arguments, one arrives at the so-called market price of risk $\lambda(t, r(t))$ and hence at a partial differential equation for zero-
bond prices,\footnote{From Lévy’s theorem it follows that $W_t(t)$ is a $\mathbb{P}$-Brownian Motion as well.} the so-called term structure equation.

$$
F_t(t, r(t)) + \lambda(t, r(t)) \sigma F_r(t, r(t)) = 0
$$

with terminal condition $F(T, r(T)) = 1$.

The Feynman–Kac\footnote{Compare Björk (2005) for further details.} formula then follows for a probabilistic interpretation of the above partial differential equation by

$$
p(t, T) = F(t, r(t)) = E^\mathbb{Q}_0 \left\{ e^{-\int_0^T r(s) ds} | r(t) \right\}
$$

with a probability measure $\mathbb{Q}$ and a stochastic process $r(t)$ with
$\mathbb{Q}$-dynamics $dr(t) = \left( a(b - r(t)) - \lambda(t, r(t)) \sigma \right) dt + \sigma dW_t(t)$

Note that observed zero-bond prices induce the market price of risk $\lambda(t, r(t))$ and therefore no obvious form or parameterization of $\lambda(t, r(t))$ exists ad hoc. However, if and only if we assume $\lambda(t, r(t)) = \lambda$, the short rate process under $\mathbb{Q}$ remains of the Vasicek-type. From standard interest rate theory (cf. e.g. Björk, 2005) it follows that $p(t, T) = e^{\langle r(t, T) - r(t, T) \rangle}$ with $A(t, T) =

\footnote{For a complete derivation cf. Graf (2008).}^3$
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