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Quantile hedging for equity-linked life insurance contracts with stochastic interest rate

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Abstract

This paper studies the problem of pricing equity-linked life insurance contracts, and also focuses on the valuation of insurance contracts with stochastic guarantee. The contracts under consideration are based on two risky assets which satisfy a two-factor jump-diffusion model: one asset is responsible for future gains, and the other one is a stochastic guarantee. As most life insurance products are long-term contracts, it is more practical to consider the problem in a stochastic interest rate environment. In our setting, the stochastic interest rate behaviour is also described by a jump-diffusion model. In addition, quantile hedging technique is developed and exploited to price such finance/insurance contracts with initial capital constraints. Explicit formulas for both the price of the contracts and the survival probability are obtained. Our results are illustrated by numerical example based on financial indexes Russell 2000 and S&P 500.

Keywords: quantile hedging, jump-diffusion, stochastic interest rate, equity-linked life insurance

1. Introduction

Equity-linked life insurance contracts have been studied since the middle of the 1970s. This type of contracts links the benefit payable at the maturity time with the market value of some reference portfolio, such as stocks, foreign currencies etc. Thus, the benefit of such contracts is uncertain while it is fixed for the traditional contracts. Compared with traditional ones, these innovative products can bring the insurance companies as well as the clients more benefit and improve the insurance companies' competitiveness in the modern financial system.

In North America and the UK, equity-linked life insurance contracts are typically provided with guarantee. Therefore, the topic of pricing equity-linked life insurance contracts with guarantee has attracted most scholars' attention. Brennan and Schwartz (1976), Boyle and Schwartz (1977) are the first papers appeared in this area. The authors decomposed the benefit of the contracts into a guaranteed

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amount and a call (put) option on the reference portfolio, then they used Black-Scholes model to evaluate the contracts. Moreover, Moeller (1998, 2001) applied the mean-variance hedging method to calculate the price of the contracts. The guarantee of the contracts in all those papers is deterministic or fixed. Ekern and Persson (1996) priced the contracts with different guarantees, fixed and stochastic using fair pricing valuation. Kirch and Melnikov (2005), Melnikov and Romanyuk (2008) also applied efficient hedging method to price the equity-linked life insurance contracts with stochastic guarantee.

Quantile hedging technique, as an imperfect hedging technique, was developed in several publications by Foellmer and Leukert (1999), and we exploit further the most important paper on this topic. It can successfully hedge the option with maximal probability in the class of self-financing strategies with restricted initial capital. This technique has been proposed by Melnikov (2004) as pricing and hedging methodology for equity-linked life insurance contracts in the Black-Scholes framework. Later it was extended by Melnikov and Skornyakova (2005) to a two factor jump-diffusion model with constant interest rate, where the second risky asset could be considered as a stochastic guarantee for the contracts.

Up till now, many research papers in the area work with a constant interest rate r. However, as insurance products are usually long-term contract, they could be more sensitive to the changes in the interest rates. Therefore, it is more practical to consider a stochastic interest rate in the financial market. Gao, et al. (2010) considered the problem of pricing equity-linked life insurance contracts by means of quantile hedging and stochastic interest rate. They studied this topic in the framework of the Black-Scholes market model driven by two independent Wiener processes and a stochastic interest rate via HJM model (See [13]). The guarantee of the contracts in their study depends on a constant rate of return g and time t.

It is well-known that discontinuous models for both the stochastic interest rate and the value of risky assets are more realistic. Extending the paper of Gao et al. (2010), we consider two risky assets S^1 and S^2 satisfying a two-factor jump-diffusion model, where the asset S^2 is less risky than S^1, and it can be seen as a stochastic guarantee of the equity-linked life insurance contract. We study the problem in the framework of Melnikov and Skornyakova (2005). But in contrast with that paper, we use a generalised HJM jump-diffusion model for the term structure of interest rate $r(t)$, which is similar to the framework of Shirakawa (1991), and Chiarella & Sklibosios (2003). Assuming independence of financial and insurance (mortality) risks, we apply quantile hedging to price equity-linked life insurance contracts with initial capital constraints.

The paper is structured as follows. In Section 2, we review jump-diffusion models and introduce the HJM term structure framework. Then we describe finance/insurance contracts under consideration. In Section3, we briefly describe quantile hedging technique and present our main pricing results. Section 4 illustrates our results with a numerical example. In Section 5 some future work is discussed. Appendix A, B and C contain technical details of proofs.

2. The financial and insurance setting

2.1. Financial model

Let $(\Omega, \mathcal{F}, (F_t)_{t\geq 0}, P)$ be a filtered probability space, where the filtration $(F_t)_{t\geq 0}$ satisfies the usual conditions and represents a flow of available information. It is supposed that all processes are adapted to this filtration. Considering a financial market with two risky assets S^1 and S^2, we use the same two factor jump-diffusion model as in Melnikov and Skornyakova (2005):

$$dS_t^i = S_{t-}^i \left(\mu_i dt + \sigma_i dW_t - \nu_i d\Pi_t \right), \quad i = 1,2 \qquad (2.1)$$