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The multi-year non-life insurance risk in the additive loss reserving model

Dorothea Diers [a,*], Marc Linde [b]

[a] University of Ulm and Provinzial NordWest Holding AG, Risk Controlling, 48131 Münster, Germany

[b] 45478 Mülheim an der Ruhr, Germany

HIGHLIGHTS

- We introduce the definition of multi-year claims development results (CDR).
- Multi-year non-life insurance risk can be expressed in terms of the multi-year CDR.
- We derive analytically closed formulae for prediction errors of the multi-year CDR.
- A case study demonstrates the applicability and usefulness of our results.

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ABSTRACT

The aim of this paper is to expand on recent contributions in the field of risk modelling for non-life insurance companies by modelling insurance risk in a multi-year context. Academic literature on non-life insurance risk to date has only considered an ultimo perspective (using traditional methods) and, more recently, a one-year perspective (for solvency purposes). However, strategic management in an insurance company requires a multi-year time horizon for economic decision making, providing the motivation for this paper.

This is the first paper to derive analytically closed formulae for multi-year non-life insurance risk in the additive loss reserving model as defined by variation of multi-year claims development results. Embedding future accident years leads to an integrated approach to quantifying multi-year risk arising from the settlement of outstanding claims (reserve risk) and future claims yet to occur (premium risk). An application study will serve to illustrate the usefulness of the new multi-year horizon.

1. Introduction

Typically, non-life insurance risk is mainly divided into reserve risk and premium risk (see Ohlsson and Lauzeningks (2009)). Reserve risk refers to outstanding payments on claims that have already occurred in prior accident years, whereas premium risk refers to claims yet to occur in future accident years. So far, both premium and reserve risk have been modelled taking the ultimo perspective. This means that the uncertainty in future claim payments is quantified up to final settlement—both for claims already occurred in the past (reserve risk) and claims yet to arise in the future (premium risk). In practice, the separation between reserve risk and premium risk is very strict, and completely different modelling approaches are even applied in some cases (see DAV-Arbeitsgruppe Interne Modelle (2008)).

The new regulatory framework, Solvency II, refers to a period of one year, so uncertainty should be quantified on future claim payments in a one-year time horizon (for examples, see Merz and Wüthrich (2008) and Ohlsson and Lauzeningks (2009)). Beyond that, a multi-year perspective is vital for practical decision making such that $m \in \mathbb{N}$ future calendar years should be taken into account. Management needs answers to questions such as: "How many years of high aggregate losses or adverse claim developments can we withstand at a certain confidence level without requiring external capital"? or "How much risk capital do we need to survive the next five years without external capital supply"? In this contribution, we will present a multi-year approach that can help answer these questions. In addition, Solvency II requires that insurance companies handle risks in their ORSA (own risk and solvency assessment) for a multi-year period (see CEIOPS (2008)); this period usually amounts to three to five years in non-life insurance. Uncertainty in future claim payments should therefore be quantified for both premium risk and reserve risk, resulting in a multi-year insurance risk consisting of multi-year premium and reserve risk over the next m calendar years (see Diers (2011)).

* Corresponding author. Tel.: +49 251 2192994.
E-mail addresses: dorothea.diers@provinzial.de, dierslr@t-online.de (D. Diers), Lindesmarc@gmx.de (M. Linde).

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The additive loss reserving method (see Merz and Wüthrich (2010)) represents a classical deterministic reserving method in non-life insurance, which yields best-estimates for future claim payments from outstanding claims. The underlying stochastic model is quite simple, allowing quantification of non-life insurance risk in terms of a multi-year claims development result. Böhm and Glaab (2006) and Merz and Wüthrich (2008) developed an analytical approach towards calculating prediction uncertainty of the one-year claims development result for the chain-ladder method. Similarly, Böhm and Glaab (2006), Mack (2009a) and Merz and Wüthrich (2010) developed an analytical approach for calculating prediction uncertainty of the one-year claims development result for the additive model.

Apart from Böhm and Glaab (2006), both cases only dealt with reserve risk while neglecting premium risk. In addition, the results only covered ultimo and one-year risk. To our knowledge, there are as yet no closed formulae for calculating the prediction uncertainty in multi-year claims development results in the additive model. The same applies to the chain-ladder model, although deriving closed analytic formulae would not be possible in this case without resorting to approximations.

The aim of this paper is to present the first analytically closed formulae derived for the multi-year non-life insurance risk in the one-year and ultimo perspectives based on additive loss reserving method. The resulting formulae for multi-year prediction errors yield a very simple representation, making them very easy to understand and implement in real life. Apart from that, we have derived an exact calculation of the prediction error of one-year claims development results in arbitrary future calendar years.

This paper makes a twofold contribution—first, we present a consistent and integrated approach to calculating premium and reserve risk for arbitrary time horizons (multi-year, one-year and ultimo) in an analytically closed approach, and second, we obtain analytical results that may be used for reference in direct comparison for simulation models. The analytical results may be used in support of multi-year risk models in strategic management decisions and the ORSA process.

The rest of the paper is structured as follows: Section 2 introduces the concept of multi-year claims development results for prior and future accident years, which is the basis for quantifying non-life reserve risk, premium risk and insurance risk in a multi-year view. Section 3 presents multi-year non-life insurance risk in the additive loss reserving model, yielding one-year, multi-year, and ultimo results. In addition, Section 3 describes the exact formula for the prediction error of future one-year claims development results in the additive loss reserving model. Section 4 applies the analytical results of the previous section to a case study. Section 5 concludes the paper.

2. The multi-year claims development result—general concept and definitions

In the following $n \in \mathbb{N}$ denotes the number of historical accident years, $m \in \mathbb{N}$ the number of future calendar years and $S_{i,k}$ the incremental payment for a single accident year $i, i \in \{1, \dots, n+m\}$, at development period $k \in \{1, \dots, n\}$. Using this convention, $C_{i,k} := \sum_{j=1}^{k} S_{i,j}$ represents the corresponding cumulative payments at development period k. We assume that each accident year is settled after n development periods latest; hence $C_{i,n} =: U_i$ is the ultimate claim amount.

At the end of period $T = n$ a claims triangle

$$\Delta_n := \left\{C_{i,k},\ 1 \le i \le n,\ 1 \le k \le n+1-i\right\}$$

of observed claims payments up to $T = n$ is available. Considering an arbitrary accident year $i, i \in \{2, \dots, n+m\}$, future claims payments $S_{i,j}, j \in \{n-i+2, \dots, n\}$ and hence the ultimate claims position U_i are generally unknown at $T = n$; thus a suitable loss reserving method needs to be applied to determine the expected future payments conditional on Δ_n—as for instance the additive loss reserving method as analyzed in our paper (see Section 3). The resulting ("opening") *best-estimate reserve* is denoted as ${}^{(n)}\widehat{R}_i$ and the estimated ultimate as ${}^{(n)}\widehat{U}_i$ respectively, where the following relationship holds:

$$ {}^{(n)}\widehat{U}_i = C_{i,n+1-i} + {}^{(n)}\widehat{R}_i. $$

The superscript n indicates that these estimates are based on the triangle data Δ_n.

New observed claims payments in calendar years $[n+1, \dots, n+m]$ lead to an amended claims triangle at the end of period $T = n+m$ (see Fig. 1):

$$\Delta_{n+m} := \left\{C_{i,k},\ 1 \le i \le n+m,\ 1 \le k \le \min(n+1+m-i, n)\right\}.$$

Only future payments $S_{i,j}, j \in \{n+2+m-i, \dots, n\}$ remain unknown. We assume that the loss reserving method initially used at $T = n$ is re-applied to claims triangle Δ_{n+m} to obtain a new ("closing") best-estimate reserve ${}^{(n+m)}\widehat{R}_i$, and hence an updated estimate for the ultimate claims position:

$$ {}^{(n+m)}\widehat{U}_i = C_{i,n+1+m-i} + {}^{(n+m)}\widehat{R}_i. $$

Extending the terminology of Merz and Wüthrich (2008) to measure the difference between the initial ultimate estimate ${}^{(n)}\widehat{U}_i$ at $T = n$ and the updated estimate ${}^{(n+m)}\widehat{U}_i$ after m further calendar years of claims information yields the following definition.

Definition 2.1 (*Multi-Year Claims Development Result for a Single Accident Year*)**.** The *m-year observable claims development result* for accident year i is defined as

$$\begin{aligned}\widehat{\mathrm{CDR}}_i^{(n\to n+m)} &:= {}^{(n)}\widehat{U}_i - {}^{(n+m)}\widehat{U}_i \\ &= {}^{(n)}\widehat{R}_i - \left(\sum_{t=1}^{\min(m,i-1)} S_{i,n+1+t-i}\right) - {}^{(n+m)}\widehat{R}_i.\end{aligned}$$

Note that a negative or positive claims development result for prior accident years $2 \le i \le n$ means an economic loss or profit over m calendar years, respectively, whereas a negative or positive claims development result for future accident years $n+1 \le i \le n+m$ implies that the corresponding estimated technical result[1] at $T = n+m$ is lower or higher than its initial estimate at $T = n$, respectively, assuming all else to be equal.

According to the model framework the final settlement of accident year i is reached in period $m = i-1$. Since there is no closing best-estimate reserve at the end of period $T = n+i-1$ anymore,

$$\widehat{\mathrm{CDR}}_i^{(n\to n+i-1)} = {}^{(n)}\widehat{R}_i - \sum_{t=1}^{i-1} S_{i,n+1+t-i}, \tag{1}$$

is called the *ultimate claims development result* and corresponds to the classical ultimo perspective. This justifies our multi-year approach as the missing link between the one-year and the ultimo perspective.

In the following, the definition of the m-year claims development result will be extended to multiple accident years.

[1] The *technical result* T_i for a single accident year i is calculated as *earned premiums* P_i – *expenses* E_i – *ultimate claims amount* U_i; see Diers (2011).