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Optimal portfolio selection with life insurance under inflation risk

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ABSTRACT

This paper investigates a continuous-time optimal consumption, investment, and life insurance decision problem of a family under inflation risk. In the financial market, there is a liquid inflation-linked index bond market which can be utilized to hedge the inflation risk. The explicit solutions are derived for constant relative risk aversion (CRRA) utility case by using martingale approach. The roles of index bond are investigated and it is verified that the index bond may have different roles depending on the market parameters. We analyze the effects of parameters on the optimal strategies with focus on the optimal demand for index bond and the optimal life insurance premium. Especially, the changes of expected inflation rate and volatility of inflation rate can have both positive and negative impacts on the life insurance premium and their quantitative impacts are considerable.

1. Introduction

For long-term financial planning of a family, not only financial risk but also inflation risk and mortality risk of the breadwinner must be considered carefully. The mortality risk of breadwinner can usually be hedged by entering insurance market. In particular, the life insurance is a typical instrument for a sudden death of breadwinner which causes financial problem of the family. On the other hand, there has been little opportunity or method for hedging inflation risk even though inflation risk affects the optimal financial decision significantly. Moreover, after expansionary monetary policy, inflation risk has become one of the most anxious factors among the side effects of the policy. So there has been high demand for inflation-linked securities and it was (partially) resolved by introducing Treasury Inflation-Protected Securities (TIPS) in the US or indexed government bonds for England, Canada, Australia, and other countries. Particularly, TIPS have been one of the most actively traded assets for recent years in the US bond market since it was introduced in 1997.

Therefore, to incorporate inflation risk and mortality risk into a financial planning, we consider a continuous-time investment/consumption decision problem of a family which includes liquid inflation-linked index bond market and mortality contingent claim (life insurance). The breadwinner of family has uncertain life time and receives labor income until a fixed retirement time if he/she is alive. During the working period of breadwinner, the family purchases life insurance to protect an absence of labor income caused by sudden death of breadwinner. It is assumed that the index bond is freely traded in the market and perfectly correlated with inflation rate. Therefore, our model is a complete market model. The object of family is to maximize their expected utility which consists of three parts: the utility from inter-temporal consumption of family, the utility from the legacy if the breadwinner dies before the retirement time, and the utility from the remaining wealth at the retirement time if the breadwinner is alive until the retirement time. For CRRA utility case, explicit solutions to the optimal consumption/investment strategies including investment in the index bond as well as the optimal life insurance premium are derived by using martingale method. Detailed explanations for the optimal strategies are also provided.

We show that the roles of index bond in the financial decision of the family are various (hedging inflation risk, alternative investment opportunity or borrowing opportunity, and portfolio diversification instrument) and the roles are determined by market parameters. For CRRA utility function, the optimal demand for index bond is composed of total capital (sum of financial capital

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and human capital) related part and the financial capital itself. The financial capital term can be interpreted as the inflation risk hedging component. The other terms represent the roles of index bond as an investment (or borrowing) opportunity and portfolio diversification. Moreover, from numerical results, we also show that the demand for index bond becomes increasing function in time near the fixed retirement time.

The effects of parameters including expected inflation rate, volatility of inflation rate, correlation between inflation risk and financial risk, risk aversion on the optimal strategies are studied. In particular, we focus on the effects of parameters on the optimal investment in index bond and the optimal life insurance premium. Obviously, the demand for index bond is increasing with respect to expected inflation rate. However, the impact of inflation rate's volatility on the demand for index bond is highly dependent on market parameters: especially the correlation between financial risk and inflation risk, and the (nominal) inflation risk premium. Depending on the signs of the correlation and the inflation risk premium, there exists a conflict between the components in the demand for index bond so that the effect of inflation rate's volatility is complicated. Furthermore, it is interesting to notice that, even though the effect of correlation also depends on market parameters, higher correlation leads to less demand for index bond in many realistic parameter sets because of reduced diversification demand.

One of our main findings is that both expected inflation rate and volatility of inflation rate can have positive and negative effects on the optimal life insurance premium. Especially, we find explicit thresholds of expected inflation rate and volatility of inflation rate where the positive effect and the negative effect coincide. If the expected inflation rate is higher (lower) than certain threshold, then the optimal life insurance premium is increasing (decreasing) in the expected inflation rate. Similarly, if the volatility of inflation rate is higher (lower) than a threshold, the optimal life insurance premium increases (decreases) as the volatility of inflation rate increases. Moreover, from the numerical results, we find that the optimal life insurance premium changes significantly depending on both expected inflation rate and volatility of inflation rate. From these observations, it can be inferred that inflation risk should be considered carefully to investigate the financial planning problem of a family with life insurance decision.

This paper is closely related to the continuous-time consumption/investment problem with life insurance introduced by Richard (1975). The succeeding papers including Pliska and Ye (2007), Ye (2007), Huang and Milevsky (2008), Huang et al. (2008), Kwak et al. (2011) and Pirvu and Zhang (2012) investigated the demand for life insurance under different market environments. Specifically, Pliska and Ye (2007) considered a wage earner whose life time is random and unbounded, and Ye (2007) included risky assets for investment. Huang and Milevsky (2008) introduced a stochastic labor income and Kwak et al. (2011) considered a financial planning problem of a family consists of parents (breadwinner) and children (other family members). Recently, Pirvu and Zhang (2012) studied a problem with mean-reverting stock return and provided closed form solutions. However, none of these papers examines the effect of inflation risk on the demand for life insurance.

Our model is also related to the literature on utility maximization problems with inflation risk. Fischer (1975) studied the demand for inflation-index bonds by solving inter-temporal optimization problem of households under inflation risk. He highlighted the role of index bond and provided a rationale for introducing an index bond market. Gong and Li (2006) considered the consumption/investment problem with index bonds and subsistence consumption constraint. Dynamic asset allocation problems including stochastic interest rate and indexed bonds are investigated by Campbell and Viceira (2001), Brennan and Xia (2002), Munk et al. (2004) and Han and Hung (2012). Especially, Campbell and Viceira (2001) and Brennan and Xia (2002) probed the importance of index bonds for long-term and conservative investment. Furthermore, Han and Hung (2012) included the DC pension plans and showed that index bonds guarantee the inflation-adjusted annuity. Siu (2011) examined the regime switching environment as well as inflation risk. However, although life insurance is important in a long-term financial planning, there is no study considering life insurance and inflation risk at the same time from the perspective of a family. Up to our knowledge, this is the first work to investigate the consumption/investment problem with inflation risk and life insurance at the same time.

This paper is organized as follows. Section 2 describes the economy we consider and Section 3 presents the optimization problem and the solutions. Section 4 examines the properties of optimal strategies, and the lifetime financial planning of the family is studied in Section 5 using numerical results. Further discussions are offered in Section 6 and Section 7 concludes the paper. Proofs can be found in appendix.

2. The economy

2.1. Financial market

We consider a continuous-time economy which consists of a financial market and an insurance market with a risk of inflation. In many cases, the typical index for inflation rate is represented by CPI (Consumer Price Index), which can be regarded as a price level process. By following Brennan and Xia (2002), it is assumed that the price level process follows a diffusion process:

$$\frac{dP(t)}{P(t)} = \mu_p(t)dt + \sigma_p dW(t), \tag{1}$$

where $\mu_p(t)$ is the expected inflation rate at time t, $\sigma_p > 0$ is the volatility of inflation rate, and $W(t)$ is a standard Brownian motion on the probability space $(\Omega, \mathcal{F}, \mathbb{P})$.

There are three kinds of assets traded in the financial market: a money market account, a stock, and an index bond which has same risk source with the price level process. Thus the inflation-linked index bond $I(t)$ follows the following stochastic process:

$$\frac{dI(t)}{I(t)} = r(t)dt + \frac{dP(t)}{P(t)} = (r(t) + \mu_p(t))dt + \sigma_p dW(t),$$

where $r(t)$ is the real interest rate at time t. In our economy, it is assumed that the inflation-linked asset (index bond) is freely traded so it fulfills the demand for hedging inflation risk. The stock follows a geometric Brownian motion with constant drift μ_s and volatility σ_s:

$$\frac{dS(t)}{S(t)} = \mu_s dt + \sigma_s dZ(t),$$

where $Z(t)$ is also a standard Brownian motion. Since the fluctuation of an inflation rate is usually affected by the outside risk factors of financial market, it is different source of risk from the financial risk. Specifically, since the inflation risk is a kind of background risk, it might have an impact on the stock return through direct or indirect ways. There are mixed empirical results for the correlation between stock return and inflation rate (Kessel, 1956; Firth, 1979; Fama, 1981; Boudhouch and Richarson, 1993; Anari and Kolari, 2001; Lee, 2010). These empirical results mainly depend on country, environment, time horizon and other factors. Therefore we presume that the correlation between financial risk and inflation risk is able to have any sign. Consequently, for $\rho \in (-1, 1)$, let us assume that

$$d\langle W, Z\rangle(t) = \rho dt,$$