Journal of Economic Dynamics & Control 35 (2011) 668–675

Contents lists available at ScienceDirect

Journal of Economic Dynamics & Control

journal homepage: www.elsevier.com/locate/jedc

Time-inconsistent preferences and social security: Revisited in continuous time

Frank N. Caliendo *

Jon M. Huntsman School of Business, Utah State University, Logan, UT 84322-3565, United States

ARTICLE INFO

Article history:
Received 14 September 2009
Accepted 6 December 2010
Available online 17 December 2010

JEL classification:
C61
H55

Keywords:
Time inconsistency
Social security

ABSTRACT

İmrohoroğlu et al. (2003) prove that it is *impossible* in a three period partial equilibrium model for social security to improve the welfare of a naive quasi-hyperbolic agent if the program has a negative net present value. This paper first generalizes their impossibility theorem to a continuous time setting and then proves analytically that no discount function exists that can rationalize a social security program with a negative net present value.

1. Introduction

Social security is commonly justified on the grounds that it prevents poverty among elderly individuals who were too shortsighted to save adequately during the working years. While this justification may seem sensible, it is problematic in the context of basic economic theory. It is well known among economists that, under basic assumptions, the standard life cycle permanent-income (LCPI) consumer with an exponential discount function cannot benefit from a social security program that has a negative net present value (below market internal rate of return), regardless of how impatient he may be. LCPI consumers will simply annuitize the loss to lifetime wealth across the entire life cycle, so that participation in such a program leads to a reduction in consumption in every period. Hence, a basic model where shortsightedness enters strictly through the exponential discount rate cannot rationalize a program with a negative net present value. Beginning with Feldstein (1985), economists have modified the LCPI model with a variety of alternative behavioral frictions in order to understand whether a social security program with a negative net present value may be rationalized under different specifications of shortsightedness.[1]

One of the leading alternatives to exponential discounting is of course hyperbolic discounting. Consumers with hyperbolic discount functions will reverse their preferences with the passage of time. A naive hyperbolic consumer standing at date 0 may intend to save part of his paycheck at date 1 for retirement, but when date 1 arrives he may not follow through with this plan. Unlike the impatient exponential consumer whose poverty in old age is all part of a grand plan first concocted when young, the hyperbolic consumer may unwittingly accumulate a suboptimal nest egg for retirement. At first glance, hyperbolic

* Tel.: +1 435 797 2963.
E-mail address: frank.caliendo@usu.edu

[1] For example, see Docquier (2002), İmrohoroğlu et al. (2003), Bucciol (2006), Cremer et al. (2008), Bucciol (2008), Kumru and Thanopoulos (2008), Findley and Caliendo (2008), Caliendo and Gahramanov (2009), and Findley and Caliendo (2009) among others.

doi:10.1016/j.jedc.2010.12.006

discounting with such time-inconsistent behavior seems like a prime setting for demonstrating the potential welfare improving role of a social security program, even one with a negative net present value.

This paper revisits Section 3 of İmrohoroğlu et al. (2003), hereafter IIJ, which shows that in partial equilibrium, naive quasi-hyperbolic consumers who do not anticipate their own time inconsistency cannot benefit from a social security program with a negative net present value.[2] They show that social security can only be welfare improving if it happens to cause distortions to factor prices in just the right way. I will call this the IIJ *impossibility theorem*, which they proved strictly for the special case of a three period model with quasi-hyperbolic preferences. The IIJ theorem is extremely important and surprising given the conventional wisdom that social security can be rationalized as a useful commitment device for hyperbolic discounters.

My purpose is to show that their impossibility theorem generalizes to a continuous time life cycle setting and the theorem holds for *any* discount function, not just a quasi-hyperbolic function. I also entertain other utility functions beyond log utility and other "tax and transfer" schemes beyond social security. Therefore, this paper strengthens the well-known IIJ impossibility theorem in three important ways: (i) The result in IIJ is not just an anomaly that comes from the quasi-hyperbolic approximation to a hyperbolic function. Nor is it even an anomaly relating to hyperbolic discounting in general. Discounting, in any form, cannot be used to rationalize a social security program with a negative net present value. (ii) The IIJ result does not depend on the coarseness of the time grid (three periods) since the present paper is cast in continuous time.[3] (iii) The IIJ result can be generalized to other well-known utility functions beyond log utility (CRRA, CARA, and quadratic) and it can be generalized to any tax and transfer scheme, in addition to social security, that rearranges the timing and magnitude of cash flows across the life cycle.

2. Setup

I will keep the model as simple as possible and free from any distractions that may confound the pure link between discounting and the optimal provision of social security. Time is continuous and is indexed by t. The individual is born and begins work at time t=0, retires at time t=T, and passes away at $t=\overline{T}$. During the working period, the individual earns wage income w, which is either consumed or saved. Consumption is $c(t)$ and the cumulative value of the savings account is $k(t)$. The savings account grows at rate r. The individual starts the life cycle with nothing in the savings account and he dies the same way. During the working years, the individual must pay social security taxes on wage income at rate θ in exchange for benefits during retirement b.

Because all discount functions other than the exponential function generate time inconsistency, we must carefully specify where in the life cycle the individual is currently standing (denoted by t=t_0), since his plan depends on where he currently stands, and we must carefully distinguish between planned behavior and actual behavior. We will derive analytical solutions in order to facilitate a proposition that exposes the inner workings of the model and the role of social security as a commitment device for time-inconsistent consumers.

2.1. *Working life* $t_0 \in [0,T]$

An individual standing at some point in the working period $t_0 \in [0,T]$ will solve the following control problem

$$\max : \int_{t_0}^{\overline{T}} F(t-t_0)\ln c(t)\, dt, \tag{1}$$

where F is any continuously differentiable, decreasing, strictly positive discount function with $F(0)$=1, subject to

$$\frac{dk(t)}{dt} = rk(t)+(1-\theta)w-c(t), \quad \text{for } t \in [t_0,T], \tag{2}$$

$$\frac{dk(t)}{dt} = rk(t)+b-c(t), \quad \text{for } t \in [T,\overline{T}], \tag{3}$$

$$k(t_0) = \text{given}, \tag{4}$$

$$k(\overline{T}) = 0. \tag{5}$$

This is a two-stage control problem due to the discrete switch in the state equation at the date of retirement.[4] For two multipliers (costate variables), $\lambda_1(t)$ defined on $[t_0,T]$ and $\lambda_2(t)$ defined on $[T,\overline{T}]$, the necessary conditions include two

[2] The naive consumer makes plans for the future and then has self-control problems in the sense that he continually fails to save as much as planned. At any instant in time the consumer makes a plan that satisfies his budget constraint and is optimal from that perspective, but in the next instant he throws out his plan and reoptimizes.

[3] Some theoretical results in life cycle models, such as the multiplicity of equilibria, are related to the time grid (I thank Jim Feigenbaum for bringing this result to my attention). So it is natural to wonder whether the IIJ theorem is somehow just a feature of the stylized model. Also, we take the time grid issue seriously because key authors in the hyperbolic literature have felt that "a complete understanding" of the hyperbolic model "will require high frequency analysis of intertemporal choices, including models embedded in continuous time" (Angeletos et al., 2001).

[4] See Tomiyama (1985) for a proof of the necessary conditions for two-stage optimal control.