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Assessing effectiveness of the various performance metrics for multi-response optimization using multiple regression ☆

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ABSTRACT

Several methods for optimization of multiple response problems using planned experimental data have been proposed in the literature. Among them, an integrated approach of multiple regression-based optimization using an overall performance criteria has become quite popular. In this article, we examine the effectiveness of five performance metrics that are used for optimization of multiple response problems. The usefulness of these performance metrics are compared with respect to a utility measure, namely, the expected total non-conformance (NC), for three experimental datasets taken from the literature. It is observed that multiple regression-based weighted signal-to-noise ratio as a performance metric is the most effective in finding an optimal solution for multiple response problems.

1. Introduction

A common approach for process optimization is a planned experimentation. Taguchi methods (Taguchi, 1990) for designing an experiment using orthogonal arrays are extremely popular among the practitioners. A large number of input process variables can be assessed to find their contribution on the output response variable using a lesser number of experimental trials. Taguchi applies the quality loss function to evaluate product quality and employs the signal-to-noise (SN) ratio with simultaneous consideration of achieving the target and reducing variability around the target value of the response variable. However, Taguchi method focuses on optimization of a single response variable only. Whereas most of the modern manufacturing processes have several response variables and the process needs to be optimized for all response characteristics.

In a multiple response optimization problem, the main objective is to find a setting combination of input process variables that would result in the optimum values of all response variables. Generally, it is very difficult to obtain such a combination, because optimum values of one response variable may lead to non-optimum values for the remaining response variables. Hence, it is desirable to find a best combination of input variables that would result in an optimal compromise of response variables. Here optimal compromise means each response variable is as close as possible to its target value and with minimum variability around that target value.

Several methods for optimization of multiple response problems using experimental data have been proposed in the literature. Some simpler methods (Pan, Wang, Wei, & Sher, 2007; Ramakrishnan & Karunamoorthy, 2006; Tai, Chen, & Wu, 1992; Tong, Chen, & Wang, 2007) use Taguchi's quality loss function and signal-to-noise (SN) ratio as the starting point of analysis procedure. The basic approach is as follows: the quality loss or SN ratio of individual responses are computed from experimental data and they are converted into an overall process performance index (PPI) and then, the factor-level combination, i.e. settings of the input variables that can optimize the PPI is determined by examining the level averages on the PPI. The advantage of the simpler PPI based approach is that it can be easily comprehended and applied even by the engineers who do not have a strong background in mathematics/statistics. However, PPI based approach cannot ensure that one or more responses will not move far away from their target values at the optimal solution.

A few integrated methods are presented by Derringer and Suich (1980), Khuri and Conlon (1981), Logothetis and Haigh (1988), Pignatiello and Joseph (1993), Tsui (1999), Wu and Hamada (2010), Wu (2005), Ch'ng et al. (2005), and Pal and Gauri (2010). In these methods, a functional relationship of each response variable with the input decision variables is established using multiple

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regression techniques, and then, the optimal solution is determined by considering an appropriately defined objective function as the performance metric for optimization. Different researchers have defined different objective functions, e.g. (i) desirability function (Derringer & Suich, 1980; Wu, 2005; Wu & Hamada, 2000), (ii) distance measure (Khuri & Conlon, 1981), (iii) multivariate (MV) loss function (Pignatiello & Joseph, 1993; Tsui, 1999), (iv) process capability index C^{*}_{pm} (Ch'ng et al., 2005), and (v) multiple regression-based weighted signal-to-noise (MRWSN) ratio (Pal & Gauri, 2010).

The aim of the present work is to examine the effectiveness of the various performance metrics that are used in the multiple regression-based methods for optimization of multi-response problems. An ideal optimal solution to a multi-response problem should result in that all the individual responses achieve their respective target value with the minimum variance around the target value. In this context, expected non-conformance (NC) of the process can be an appropriate measure of the goodness of an optimal solution. So expected NC at the optimal solution is considered here as the utility measure of a performance metric. It may be noted that expected NC can be computed only if the specifications for the responses are known. For the comparison of the usefulness of the various performance metrics, therefore, three experimental datasets with known specifications for the responses are analyzed using various multiple regression-based multi-response optimization procedures.

The article is organized as follows: the second section provides a brief literature review on the existing methodologies for dealing with the multi-response optimization problems. The third section outlines the multiple regression-based optimization procedure using different performance metrics. The fourth section describes the utility functions, namely, expected process NC and the procedure for comparison of different performance metrics. In Section 5, we consider three sets of experimental data for analysis and comparison. We conclude in Section 6.

2. Literature survey

The analysis of multiple response experiments has received high attention over the last three decades. Derringer and Suich (1980) and Khuri and Conlon (1981) proposed the multiple regression-based approach to solve multiple response problems using some performance metric. Derringer and Suich (1980) used a desirability function, first defined by Harrington (1965), as a metric for optimization of multiple response variables. Each response is converted into an individual desirability function and then an overall desirability function is defined as the geometric mean of the individual response desirabilities. Khuri and Conlon (1981) developed a regression-based optimization method using a distance metric. This distance metric uses the squared deviations of the response variables from their targets and then standardizes these deviations by the variance of the response variables. Logothetis and Haigh (1988) proposed an optimization technique by using multiple regression and the linear programming approach. In the linear programming model, one response variable is selected as primary objective function and is optimized by using other response variables in the constraints criteria. Pignatiello and Joseph (1993) present a multiple regression technique based on the criteria of minimizing the expected value of a multivariate loss function. Their procedure assumes that the responses follow a multivariate normal distribution, are nominal-the-best (NTB) type and follow an additive model. Tsui (1999) extended Pignatiello and Joseph's procedure to situations where responses may be smaller-the-better (STB) type or larger-the-better (LTB) type of quality characteristics. Wu and Hamada (2000), and later Wu (2005) suggested using double-exponential desirability function and proposed a regression-based approach for optimization of correlated multiple quality characteristics. Pal and Gauri (2010) proposed an integrated approach using multiple regression technique and weighted signal-to-noise ratio for multiple response optimization. In this method, the weighted SN ratio is maximized ensuring that all the quality characteristics (responses) are very close to their respective target values and with minimum variability around the target values.

A few simpler methods were developed using a metric known as process performance index. These methods include weighted signal-to-noise ratio (WSN) (Tai et al., 1992), multi-response signal-to-noise ratio (MRSN) (Ramakrishnan & Karunamoorthy, 2006), grey relational analysis (GRA) (Pan et al., 2007), and VIKOR (*VlseKriterijumska Optimizacija I Kompromisno Resenje in Serbian*) (Tong et al., 2007) methods. In these approaches, the quality loss or SN ratio of individual responses are converted first into an overall process performance index (PPI) and then, the factor-level combination, i.e. settings of the input variables that can optimize the PPI is determined examining the level averages on the PPI. These methods are well acceptable to the practitioners because of their simplicity. However, one limitation of those approaches, except the VIKOR method (Tong et al., 2007), is that the optimal factor-level combination often result in one or more individual responses to move far away from their target values which are not desirable.

Another group of researchers have attempted to integrate some of the advanced techniques such as principal component analysis (PCA) with the simpler approaches, i.e. PPI based approaches. Su and Tong (1997) and Liao (2006) have applied principal component analysis (PCA) and then, computed the PPI (named differently by the authors) using one or more principal components instead of the original response variables. While Su and Tong (1997) have used the first principal component only, Liao (2006) have utilized all the principal components and his proposed approach is known as weighted principal component (WPC) method. Tong et al. (2005) integrated PCA with the technique for order preference by similarity to ideal solution (TOPSIS).

Some researchers like Su and Hsieh (1998), Tong and Hsieh (2000), Hsi, Tsai, Wu, and Tzuang (1999) and Chiang and Su (2003) have found that the techniques of artificial intelligence can be effectively used for process optimization. In these approaches, parameters can be set optimally but nothing can be learned about the relationship between the control factors and the responses, and so do not help engineers to learn efficient engineering experiences during process optimization.

A significant advancement has been made with the introduction of response surface methodology (RSM) for optimization of multiple response problems. RSM typically involves experimental design, regression models and optimization. Regression models are built based on the data collected in the experimental design and then optimization is done using the regression models. Vining and Myers (1990) first introduced dual response surface optimization technique to solve joint optimization problem of mean and variance of a single response variable in Taguchi's experimental design. Del Castillo and Montgomery (1993) and Lin and Tu (1995) have contributed significantly to the development of dual response surface optimization. In multiple response surface optimization, researchers seek to optimize simultaneously the mean and variance of several response variables. Ch'ng et al. (2005) have proposed the usage of RSM and C^{*}_{pm} metric to optimize multiple response variables.

Multiple regression-based optimization methods are essentially a part of response surface methodology. RSM techniques are based on a series of experimentation which are different from Taguchi's experimental design using orthogonal arrays. RSM tries to obtain an optimal solution using the entire range of values of input vari-