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On the optimal size of Social Security in the presence of a stock market

Marten Hillebrand *

Karlsruhe Institute of Technology (KIT), Department of Economics and Business Engineering, Kollegium IV am Schloß, D-76128 Karlsruhe, Germany

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ABSTRACT

The paper develops a stylized overlapping generations economy with random production and a stock market. The impact of a Social Security system on production, asset markets, and consumer welfare is analyzed. It is shown that any reduction in the contribution rate fosters capital accumulation and increases asset prices, wages, and production output. Different welfare criteria are applied to determine the optimal size of Social Security. It is shown that there exists a unique contribution rate which is long-run optimal, socially optimal, and time-consistent in the sense that no generation has an incentive to change it.

1. Introduction

Do stock market investments provide a better form of retirement provision than Social Security? This question plays an important role in the debate about the design and reform of the US and other Social Security systems. People who affirm it typically argue that empirical returns on stock market investments are in general higher than those earned by the Social Security system. As a consequence, they favor a privatized system with a reduced share of public pension benefits and a larger share of private investment for retirement in the stock market. Opponents of such a reform, however, argue that stock market investments are subjected to capital market risks. Hence, the proposed privatization would lead to an unfavorable increase of the risk to which retirement incomes are exposed.

The latter argument stresses the importance to incorporate the role of risk and uncertainty when performing a theoretical study of Social Security. In addition, several types of interaction between the Social Security system and other economic variables are known to be important in the literature. In this regard, as first shown by Feldstein (1974) the presence of Social Security affects consumers' savings behavior and crowds out private investment which harms the accumulation of capital. By the same mechanism, changes in the parameters of the Social Security system will in general affect (equilibrium) prices on asset markets and, therefore, stock market returns. This insight makes apparent that a simple static comparison of stock market and Social Security returns should be done with care and calls for a framework that incorporates the mutual interactions between Social Security, asset markets, and the production process.

The existing studies in the literature account for these interactions in different ways and to different degrees of generality. A large body of the existing models takes the production process as exogenous and studies the efficiency properties of equilibria in a stochastic overlapping generations (OLG) framework. Examples of such pure exchange models may be found in Chattopadhyay and Gottardi (1999), Demange and Laroque (1999), or Peled (1984). The employed welfare concepts are usually those of ex-ante optimality (EAO) and conditional Pareto optimality (CPO); cf. Demange (2002) for a survey. These concepts generalize the traditional Pareto criterion to stochastic OLG economies. For a study of Social Security, however, the framework with an exogenous production process seems inappropriate as it does not incorporate the adverse effect on capital accumulation exhibited above.

An endogenous description of capital accumulation and the production process in a stochastic OLG framework is developed in Wang (1993). His model was extended by Hauenschild (2002) to show that the adverse effect of Social Security on capital accumulation extends to a general class of stochastic OLG economies. The importance of this insight derives from the fact that the first welfare theorem need not hold in OLG economies and equilibria may be inefficient due to an overaccumulation of capital. In such a situation, the introduction of Social Security may lead to a welfare improvement by implementing a dynamically

* Tel.: +49 0 721 608 5667; fax: +49 0 721 608 3082.
E-mail address: marten.hillebrand@kit.edu.

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efficient allocation. A second potential welfare gain through the presence of Social Security is stressed and analyzed in Krüger and Kübler (2006) and Gottardi and Kübler (2008). They show that in the presence of incomplete financial markets, Social Security may lead to a welfare improvement due to an improved risk sharing between generations. Further recent studies of these risk-sharing effects may be found in Bohn (2009) and Olovsson (2010).

In most of these models the notion of a stock market corresponds to a market for real capital which is built one for one from previous output. Abel (2003) generalizes this structure by assuming a non-linear capital adjustment technology to obtain a non-trivial capital price process. This modification allows him to analyze the impact of changes in the population structure on capital/stock prices and consumer welfare under different Social Security systems. Other assets such as bonds, etc. do not exist in his model. Their existence typically requires a multiperiod OLG structure as in Krüger and Kübler (2006) or Olovsson (2010). In this case, however, the derivation of analytical results and closed form solutions seems impossible.

The present paper seeks to contribute to the discussion motivated above by studying the welfare effects and optimal size of Social Security in the presence of a stock market. The study complements the above approaches with respect to both the underlying framework as well as the employed welfare concepts. As for the framework, the assumptions made with respect to technologies and consumer preferences are similar to Abel (2003) sacrificing full generality to obtain closed form solutions and analytical results. The main difference to his and other existing studies is the assumption that capital is installed in (and owned by) firms and cannot be traded between generations directly. Instead, the notion of a stock market corresponds to a market where claims on the firms' dividend payments are traded between generations. In addition, a bond market exist which is used by the firm to finance its capital investment and provides a riskless investment possibility to consumers. This structure was developed in Dechert and Yamamoto (1992) and Magill and Quinzii (2003) and is compatible with two-period lived consumers. Worth noting is the fact that stock market investments are essentially unproductive in this setting which stands in stark contrast to their role, e.g., in Abel (2003). The present paper reveals how crucial this difference is for the welfare effects of Social Security.

With reference to the discussion of whether privatizing Social Security is favorable, the main goal of the paper is to determine the optimal size of the system corresponding to an optimal contribution level. For this purpose, the paper employs several welfare concepts including *long-run optimality*, *social optimality*, and *time consistency*. Unlike the aforementioned Pareto criteria, these concepts incorporate the trade-off between the interests of different generations and permit to identify a unique optimal allocation. Existing studies which also employ (versions of) these concepts may be found e.g., in Abel (2003), Bohn (2009), Demange and Laroque (2000) and Hillebrand (2011), who conducts a related study in a model with endogenous labor supply and no asset markets.

The paper is organized as follows. Section 2 introduces the underlying model describing the behavior of consumers and the firm. Existence of equilibrium and the equilibrium effects of Social Security are studied in Sections 3 and 4. Sections 5–7 employ the welfare concepts described above to determine the optimal size of Social Security. Section 8 draws some conclusions, proofs for all results are placed in the Appendix.

2. The model

Population and Social Security. The consumer side of the economy consists of overlapping generations of homogeneous consumers who live for two periods. The index $j \in \{y, o\}$ identifies the young and old generation in each period. Each young consumer is endowed with one unit of labor time supplied inelastically to the labor market. Old consumers are retired and do not supply labor. The number of young consumers born in each period grows at constant rate $\eta > -1$ such that aggregate labor force (equivalently, the number of young consumers) evolves as $L_t^S = (1+\eta)L_{t-1}^S$, $t > 0$.

There exists a single consumption commodity in the economy which serves as numeraire such that all prices and payments are denominated in units of the consumption good. Denote by $w_t > 0$ the real wage per labor unit at time t to be determined below. As in Hauenschild (2002), Social Security is modeled as a redistributive pure pay-as-you-go system which is characterized by a constant proportional income tax $\tau \in]\tau_{\min}, 1[$ on a young consumer's labor income. The lower bound $\tau_{\min} < 0$ will be determined below. All revenues are distributed lump-sum to the old. A negative contribution rate corresponds to a transfer from old to young consumers. Given τ, the non-capital incomes of young and old consumers are determined from the real wage at time t as

$$e_t^y = (1-\tau)w_t \quad \text{and} \quad e_t^o = (1+\eta)\tau w_t. \tag{1}$$

Assets and portfolios. To invest their income e_t^y, young consumers face two investment possibilities corresponding to different assets. The first asset is a one-period lived bond which can be purchased at a price of unity at time t and pays a gross return $R_t > 0$ per unit in the following period. Since R_t is determined at time t, the bond provides a riskless investment possibility between any two consecutive periods. The second asset is given by retradeable shares (stocks) of a firm which are traded at price P_t and pay a dividend D_t (per share) prior to trading at time t. Neither the dividend payment D_{t+1} nor the selling price P_{t+1} are known at time t such that the return on any stock investment is uncertain. In the sequel we normalize asset prices and dividends by setting $p_t := \frac{P_t}{L_t^S}$ and $d_t := \frac{D_t}{L_t^S}$. The total number of shares is normalized to one and is initially held by old consumers in each period t.

Let $(b_t, \tilde{z}_t) \in \mathbb{R}_+^2$ be the investment of a young consumer in period t with b_t denoting the number of bonds and $\tilde{z}_t$ the number of shares purchased. As the equilibrium stock holdings of each young consumer at time t will be given by $z_t = L_t^{S-1}$, it will be convenient to measure stock investments relative to the equilibrium portfolio by setting $x_t := \frac{\tilde{z}_t}{z_t} = \tilde{z}_t L_t^S$ and to denote the consumer's portfolio equivalently as $(b_t, x_t) \in \mathbb{R}_+^2$.

Consumer demand. Given her current net income e_t^y determined by (1), the current stock price p_t, and the return on bonds R_t, a young consumer's choice of a portfolio (b, x) implies consumption in period t as

$$c_t^y = e_t^y - b - p_t x \tag{2}$$

and next period's consumption corresponds to the random variable

$$c_{t+1}^o = R_t b + [1+\eta][\tau w_{t+1} + (p_{t+1} + d_{t+1})x]. \tag{3}$$

Uncertainty in next period's consumption enters through next period's wage w_{t+1} and prices p_{t+1} and dividends d_{t+1} of the risky asset. These quantities are treated as given random variables at time t. To evaluate different portfolio decisions, young consumers have preferences over alternative consumption plans (c_t^y, c_{t+1}^o). These preferences possess an expected utility representation with von-Neumann–Morgenstern utility function

$$(c^y, c^o) \longmapsto u(c^y) + \gamma u(c^o), \qquad u(c) := \ln c, \quad \gamma > 0. \tag{4}$$

Each young consumer chooses a portfolio which maximizes her expected utility of lifetime consumption. The corresponding maximization problem reads

$$\max_{b,x \geq 0} \Big\{ \ln\big(e_t^y - b - p_t x\big) + \gamma\, \mathbb{E}_t\big[\ln\big(R_t\, b + [1+\eta][\tau\, w_{t+1} + (p_{t+1} + d_{t+1})\, x]\big)\big] \,|\, b + p_t\, x \leq e_t^y \Big\}. \tag{5}$$