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Monte Carlo computation of optimal portfolios in complete markets☆

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Abstract

We introduce a method that relies exclusively on Monte Carlo simulation in order to compute numerically optimal portfolio values for utility maximization problems. Our method is quite general and only requires complete markets and knowledge of the dynamics of the security processes. It can be applied regardless of the number of factors and of whether the agent derives utility from intertemporal consumption, terminal wealth or both. We also perform some comparative statics analysis. Our comparative statics show that risk aversion has by far the greatest influence on the value of the optimal portfolio. © 2002 Elsevier Science B.V. All rights reserved.

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1. Introduction

The derivation of the optimal portfolio of a rational investor is a central problem in asset pricing. Although the interest of closed-form solutions that would allow to derive equilibrium implications is obvious, the increase in computational power along with the lack of closed-form solutions for many interesting cases have triggered an interest in numerical methods as a possible answer to the problem. In this paper, we suggest

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a method purely based on Monte Carlo simulation that allows to solve the problem in complete markets.

Merton (1971) introduced a methodology to attack the problem of a rational investor with time additive preferences that chooses how to allocate her wealth between consumption and the existing securities. In his setting, computation of the optimal consumption and investment strategies requires the solution of a partial differential equation (PDE). However, that PDE only has a closed-form solution in a handful of cases. Karatzas et al. (1987) and Cox and Huang (1989) introduced martingale methods to solve the problem of an utility optimizing investor. Martingale methods allow to consider more general settings than dynamic programming methods and in general allow to compute the optimal consumption policy of the investor. But optimal portfolios, in general, cannot be computed in closed form when martingale methods are used.

A large number of papers have recently undertaken the problem of computation of optimal portfolios. Campbell and Viceira (1999) and Barberis (2000) use numerical approximations to find optimal portfolios in a discrete time setting. In continuous time, Kim and Omberg (1996) solve the PDE in closed-form for a specific parametrization of the model. Liu (1998) finds a closed-form solution for a general class of parameterizations for an agent whose utility depends only on terminal wealth (TW). Wachter (1999) solves also a specific case but one that allows for intertemporal consumption (IC). Brennan et al. (1997) and Xia (1999) solve numerically the PDE also for specific (but more general) parameterizations of the utility function. Finally, Detemple et al. (1999) compute the Malliavin derivatives of the processes and then use Monte Carlo simulation in order to retrieve the optimal portfolio. In this paper we introduce a pure Monte Carlo simulation approach, very easy to implement and that can be applied whenever two conditions are met (this two conditions are also required by the method introduced in Detemple et al. (1999):

- Markets are complete, that is, the number of non-redundant stocks and the number of Brownian motion processes that explain the uncertainty of the economy are equal.
- We know the dynamics of all the processes involved (the *expanded opportunity set* is Markovian).

The method we introduce here can be applied to any type of time additive utility function and any parametrization of the security processes, regardless of whether the agent derives utility from final wealth, IC or both, and regardless of the number of Brownian motion processes that explain the uncertainty of the economy. The advantage of Monte Carlo simulation is that it is very easy to implement and converges reasonably fast. Monte Carlo simulation has been increasingly popular for pricing derivatives since its introduction in finance by Boyle (1977). However, it had not been considered as a tool to solve optimal portfolios until the work of Detemple et al. (1999).

Our method uses the fact that the optimal portfolio of the investor is part of the instantaneous standard deviation of the optimal wealth process (in a contemporaneous paper, Lioui and Poncet (2001) use that fact in order to derive optimal portfolios in a stochastic interest rate setting). The latter can be computed by finding second moments (the expected value of the squared change in the wealth level), and, therefore,