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# Estimating the effective degrees of freedom in univariate multiple regression analysis

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**Abstract**

The general linear model provides the most widely applied statistical framework for analyzing functional MRI (fMRI) data. With the increasing temporal resolution of recent scanning protocols, and more elaborate data preprocessing schemes, data independency is no longer a valid assumption. In this paper, we revise the statistical background of the general linear model in the presence of temporal autocorrelations. First, when detecting the activation signal, we explicitly account for the temporal autocorrelation structure, which yields a generalized $F$-test and the associated corrected (or effective) degrees of freedom (DOF). The proposed approach is data driven and thus independent of any specific preprocessing method. Then, for event-related protocols, we propose a new model for the temporal autocorrelations (''damped oscillator'' model) and compare this model to another, previously used in the field (first-order autoregressive model, or AR(1) model). In the case of long fMRI time series, an efficient approximation for the number of effective DOF is provided for both models. Finally, the validity of our approach is assessed using simulated and real fMRI data and is compared with more conventional methods. 

*Keywords:* Functional MRI; Effective degrees of freedom; Multiple linear regression; Temporal autocorrelations

## 1. Introduction

In their seminal papers on analyzing fMRI time series data using multiple regression, Friston et al. (1995a) and Worsley and Friston (1995) elucidated the importance of taking autocorrelations in the statistical tests into account. They suggested modifying the number of degrees of freedom (DOF) associated with the statistical tests performed to detect significantly activated voxels. This correction was found to be necessary, because regressors were convolved with a Gaussian smoothing kernel in the temporal domain, thus correlating the data. The Gaussian kernel was matched to the shape of the hemodynamic response, and introduced in order to obtain a better signal-to-noise ratio.

In the formulation given by Worsley and Friston (1995), the autocorrelations of the filtered fMRI data are approximated by the properties of the matched Gaussian filter, i.e. properties of the unfiltered data are ignored. This approximation is valid if the autocorrelations in the unfiltered data are low, and filter properties are matched to the signal of interest. Since the temporal filter is uniformly applied in the spatial domain to all voxels in the fMRI data set, it follows that a *spatially uniform* assumption about autocorrelations is implied. With increasing knowledge of the signal properties of fMRI data, assumptions about spatially homogeneous hemodynamic responses and spatially homogeneous noise properties no longer hold. More efficient methods for separating the fMRI signal from physiological and system noise were proposed as a preprocessing step (Biswal et al., 1996; Buonocore and

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Maddock, 1997; Hu et al., 1995; Kruggel et al., 1999), which renders a second smoothing step unnecessary. However, this raises the issue of how to correct for the enhanced autocorrelations introduced by these filters in the subsequent signal detection step. It appears more attractive to take the properties of the (filtered) data into account directly in the statistical evaluation. Consequently, the number of DOF used for statistical inference must be computed voxelwise and adapted to the properties of the preprocessing filter.

In addition, there are computational issues to discuss. Today's MR scanner hardware make volume acquisition in subsecond intervals possible, and experimental designs of more than 2000 time steps are common. It is therefore desirable to design methods of statistical analysis for which the computation time is *not* a limiting factor to their routine use.

In this paper, we start by presenting a rather scholastic review of the statistical background of handling autocorrelations in the general linear model. While comprehensive treaties of this problem appeared early in the statistical literature (Cochrane and Orcutt, 1949; Durbin and Watson, 1950, 1951), formulations targeted to analyze fMRI data appear only scattered throughout specialized textbooks (Graybill, 1983; Johnson and Wichern, 1988; Rencher, 1995; Lange and Zeger, 1997; Moonen and Bandettini, 1999), and deserve a compact review. We derive a generalized $F$-test for detecting significantly activated areas and explicitly compute the number of effective DOF associated with this $F$-test. The test is independent from a smoothing matrix introduced in the design and thus allows the application of any suitable filter or signal restoration algorithm in a preprocessing step. However, it depends on the temporal autocorrelation structure of the data. Therefore, we propose a new autocorrelation model (''damped oscillator'' model) suitable for event-related studies and compare it with the more conventional AR(1) model. Optimizations and approximations are discussed which make a statistically valid handling of autocorrelations computationally tractable. We then validate the proposed autocorrelation model by studying semi-synthetic fMRI data filtered by different preprocessing algorithms. Finally, a realistic example demonstrates the consequences of this approach in the signal detection step of fMRI data analysis.

## 2. Theoretical background

### 2.1. General linear model

Univariate multiple regression represents one application of the general linear model (Rencher, 1995), which is a powerful tool for assessing relationships between the fMRI signal and experimental factors (Friston et al., 1995b; see Moonen and Bandettini, 1999, for a thorough discussion). Denote by $\boldsymbol{y}$ an $N$-vector of fMRI time series data (usually preprocessed data) and by $\boldsymbol{x}$ an $(N,P)$ matrix assumed to have full rank. Each of the $P$ columns of $\boldsymbol{x}$ is called a ''regressor'', which is either determined by the experimental design (''regressors of interest'') or represents confounds (''dummy regressors''). The general linear model relates observations with the model via a $P$-vector of regression coefficients $\boldsymbol{\beta}$,

$$\boldsymbol{y} = \boldsymbol{x}\boldsymbol{\beta} + \boldsymbol{\epsilon}, \quad (1)$$

where $\boldsymbol{\epsilon}$ corresponds to an $N$-vector of error terms. Let us assume that the errors are restricted to the observations $\boldsymbol{y}$, i.e. there are no errors in the regressors $\boldsymbol{x}$. The process generating $\boldsymbol{\epsilon}$ is assumed to be stationary in time. Errors are assumed to be normally distributed and correlated, following $\mathcal{N}(0,\sigma^2\boldsymbol{V})$, where $\sigma^2$ is a constant and $\boldsymbol{V}$ corresponds to the autocorrelation matrix. In the following discussion, the actual structure of $\boldsymbol{x}$ is not relevant: the stimulus function may have been convolved with a hemodynamic modulation function using pre-defined (Friston et al., 1995b) or data-driven parameters (Rajapakse et al., 1998).

Solving the general linear model consists of deciding whether or not $\boldsymbol{y}$ represents a brain activation signal by estimating the regression coefficients $\hat{\boldsymbol{\beta}}$ and determining, using a statistical test, whether or not they contribute significantly to predicting the signal $\boldsymbol{y}$.

### 2.2. Solving the general linear model using ordinary least squares

The most widely used approach for solving the general linear model consists of estimating the regression coefficients $\hat{\boldsymbol{\beta}}$ using ordinary least squares (OLS),

$$\hat{\boldsymbol{\beta}} = (\boldsymbol{x}^{\mathrm{T}}\boldsymbol{x})^{-1}\boldsymbol{x}^{\mathrm{T}}\boldsymbol{y}, \quad (2)$$

where $\boldsymbol{x}\hat{\boldsymbol{\beta}}$ represents the estimated signal and $\hat{\boldsymbol{\epsilon}} = \boldsymbol{y} - \boldsymbol{x}\hat{\boldsymbol{\beta}}$ is the residual, i.e. the error between the actual and the estimated time series.[1] The estimated regression coefficients $\hat{\boldsymbol{\beta}}$ are unbiased, but do not have minimum variance because the OLS approach assumes that errors are independently distributed.

A number $Q$ of regressors may be tested for having a significant influence on predicting the time series $\boldsymbol{y}$ under the null hypothesis $H_0 : \hat{\boldsymbol{\beta}}_1 = 0, \ldots, \hat{\boldsymbol{\beta}}_Q = 0$. For doing so, a reduced model $\boldsymbol{y} = \boldsymbol{x}_{\mathrm{r}}\boldsymbol{\beta}_{\mathrm{r}} + \boldsymbol{\epsilon}_{\mathrm{r}}$ has to be solved, where $\boldsymbol{x}_{\mathrm{r}}$ represents $\boldsymbol{x}$ in which the $Q$ studied factors have been removed. The $\hat{\boldsymbol{\beta}}_{\mathrm{r}}$ are estimated using OLS and the following sums of squares are computed:

- The sum of squares due to the residuals $\mathrm{SSR} = \hat{\boldsymbol{\epsilon}}^{\mathrm{T}}\hat{\boldsymbol{\epsilon}} = \boldsymbol{y}^{\mathrm{T}}\boldsymbol{y} - \hat{\boldsymbol{\beta}}^{\mathrm{T}}\boldsymbol{x}^{\mathrm{T}}\boldsymbol{y}$, with an associated number of effective DOF $\nu_{\mathrm{SSR}}$.
- The sum of squares due to the model $\mathrm{SSH} = \hat{\boldsymbol{\epsilon}}_{\mathrm{r}}^{\mathrm{T}}\hat{\boldsymbol{\epsilon}}_{\mathrm{r}} -$

[1] If $\boldsymbol{x}$ does not have full rank, then $\hat{\boldsymbol{\beta}} = (\boldsymbol{x}^{\mathrm{T}}\boldsymbol{x})^{-}\boldsymbol{x}^{\mathrm{T}}\boldsymbol{y}$, where ''$^{-}$'' denotes the generalized inverse.