



Approximate solution to Fredholm integral equations using linear regression and applications to heat and mass transfer

Yiannos Ioannou^a, Marios M. Fyrillas^{b,*}, Charalabos Doumanidis^a

^a Department of Mechanical Engineering, University of Cyprus, 1678 Nicosia, Cyprus

^b Department of Mechanical Engineering, Frederick University, 1303 Nicosia, Cyprus

ARTICLE INFO

Article history:

Received 10 June 2011

Accepted 8 February 2012

Available online 29 March 2012

Keywords:

Heat/mass transfer

Integral equations

Regression analysis

Matched asymptotic expansions

ABSTRACT

In this work we develop improved asymptotic solutions to one-dimensional Fredholm integral equations of the first kind using linear regression. For the cases under consideration the unknown function is the flux distribution along a strip, and the integral equation depends on a parameter or a number of parameters, i.e. the Péclet number, the Biot number, the dimensionless length scales etc. It is assumed that asymptotic solutions, with respect to the parameters, are available. We show that the asymptotic solutions can be improved and extended by relaxing the coefficients associated with them and applying regression analysis to yield best-fit coefficients. The asymptotic solutions may even be combined to obtain a matched asymptotic expansion. Explicit expressions for the coefficients, which can depend on a number of parameters, are obtained using regression analysis, i.e. by creating a variational principle for the Fredholm Integral Equation and employing the least squares method. The resulting expression, although it provides an approximate solution to the flux distribution, it is explicit and estimates accurately the overall transport rate.

© 2012 Elsevier Ltd. All rights reserved.

1. Introduction

In general, through a class of numerical methods known under the aliases boundary integral, boundary element, boundary integral-equation, panel and Green's function methods, the classical scalar transport process in a medium [1] can be described by a kernel function (Green's function G), which depends on a number of parameters (P_i) characterizing the material/process conditions, and the flux distribution (q) [2,3]. The resulting equation is a convolution integral equation known as a Fredholm integral equation [4]. This class of numerical methods offers the natural choice for inverse transport problems as it combines numerical simplicity and accuracy. Regarding the former, the reduction of the differential equation to an integral equation over the boundary reduces the dimensionality, hence the complexity, of the problem. Furthermore, the integral representation allows the consideration of an infinite-domain, direct calculation of the concentration gradient, and de-singularization of the singular points which translate into a high degree of accuracy [5–12,14–16,21]. Although the resulting integral equation is linear, it is notoriously difficult to solve and explicit solutions are only available for some special

forms of the kernel [31]. Techniques for obtaining asymptotic solutions have appeared in the literature [8–12,42,43,14,17–20] however, it is often difficult to proceed to higher order or to obtain a matched asymptotic expansion.

In natural and technological processes described by scalar (mass, energy, charge, etc.) transport phenomena, inverse transport problems have recently re-gained the keen interest of the scientific and engineering communities. Such inverse transport formulations seek the requisite input flux distribution (q), as well as the total flux (Q) required, in order to obtain a specified density distribution in the continuum (e.g. concentration, temperature, potential, etc.). The reason for this rekindled interest is due to both technical and mathematical progress in transport research: Technological means have become available over the past few decades for control of the flux distribution q , via high-bandwidth continuous scanning sources of lower dimensionality (e.g. robotic or servomechanism-guided plasma or laser heat torches [22–24], high-density infrared lamp strips [25], scanned electron or ion implantation beams [26], etc.). In addition, highly localized discrete stationary sources, at dimensions down to the nanoscale (e.g. nanoheaters [27,28]) and in custom-designed distributions or addressable multiplexed array configurations, are presently introduced for precise implementation of transport actuation in a variety of applications (self-heating, self-repaired materials, etc.).

From the computational perspective, aside from off-line numerical simulation formulations (finite difference, finite element,

* Corresponding author.

E-mail addresses: ioannouy@gmail.com (Y. Ioannou), m.fyrillas@gmail.com (M.M. Fyrillas), cdoumani@ucy.ac.cy (C. Doumanidis).

boundary value methods, etc.) and useful software tools that have become ubiquitous in engineering practice, there is also renewed attention to analytical techniques for inverse transport problems. Analytical solutions afford unique physical insights on the explicit effects of transport parameters, as well as computational efficiencies enabling their in-process technical implementation, in conjunction with the new actuation technologies mentioned above. Typical such analytical methods involve an optimization approach in approximating the desired density distribution in the continua by the action of combined/distributed elementary influxes (e.g. via a Green–Galerkin technique [29]). However, in an effort to avoid real-time numerical computation costs as much as possible during transport applications, there is a need for efficient optimization in the solution of the Fredholm integral equations (describing linear, stationary transport in a low-dimensionality continuum) by utilization of known analytical distribution functions in the asymptotic limits of the process parameters, i.e. corresponding to simpler, ideal physical problems.

Motivated by such a necessity, in the next Section 2 we develop a method that can improve the existing asymptotic solutions of Fredholm integral equations of the first kind, through a least squares regression method. In the following Section 3, the method is implemented in a number of examples motivated from classical problems in heat and mass transfer.

2. Development of the least squares regression method

In this section, we develop a method that can improve and extend existing asymptotic solutions of Fredholm integral equations of the first kind. The limitations and assumptions, associated with the applicability of the method, are outlined during the development of the method that follows.

Consider a one-dimensional Fredholm integral equation of the first kind [4,7–12,14,30]. Without loss of generality we can assume the form

$$T_s[x] = \int_0^1 q[x'; P] G[x, x'; P] dx', \tag{1}$$

where $G[x; P]$ is the kernel i.e. the Green's function associated with the problem, $q[x; P]$ is the unknown function i.e. the flux distribution, $T_s[x]$ is a known function i.e. the temperature distribution along a strip, and P is a dimensionless parameter (or parameters P_i) associated with the physical problem, i.e. the Péclet number, the Biot number, the length scales, etc. Here, we should point out that explicit solution for such equations are only available for some special forms of the kernel [31].

To proceed, we further assume that explicit forms of the unknown function $q[x; P]$ are available in the asymptotic limits of the parameter P , which we denote as $q_0[x; P \ll 1]$ and $q_\infty[x; P \gg 1]$. In heat and mass transfer applications the functions q_0 and q_∞ take the form [9–12,14]

$$q_0[x; P \ll 1] = \frac{a_0[P]}{f_0[x]} \tag{2}$$

and

$$q_\infty[x; P \gg 1] = \frac{a_\infty[P]}{f_\infty[x]}, \tag{3}$$

where $f_0[x]$ and $f_\infty[x]$ are explicit functions of x , and $a_0[P]$, $a_\infty[P]$ are explicit functions of the parameter P . The expressions for q_0 and q_∞ can be obtained through boundary layer analysis [9,32], the Wiener–Hopf technique [7,8,14], by asymptotic analysis [9–12], or combination of the above techniques. We are interested to obtain an approximate expression for the flux distribution $q[x; P]$ for arbitrary

values of the parameter P , through which we can also obtain the total transport rate

$$Q[P] = - \int_0^1 q[x; P] dx. \tag{4}$$

The improved asymptotic solution that follows is motivated by the fact that numerical calculations demonstrate the persistence of the asymptotic functional relations $1/f_0[x]$ and $1/f_\infty[x]$ beyond the region of validity implied by the respective asymptotic analysis. Hence, this suggests a regression analysis where the solution is approximated as the sum of the two asymptotic solutions (2) and (3),

$$q_{ls}[x; P] = \frac{a_{0m}[P]}{f_0[x]} + \frac{a_{\infty m}[P]}{f_\infty[x]}, \tag{5}$$

where however, the coefficients $a_{0m}[P]$ and $a_{\infty m}[P]$ are now free expressions [33–36,20] that are going to be estimated using a variational principle. The integral equation may now be transformed into a regression problem [37,38]. We proceed using least squares, however one might attempt to use collocation, minimax or some other method. Here we should point out that the particular form of q_{ls} along with the choice of regression method may offer opportunities for addressing integral equations using numerical minimization techniques.

Applying least squares regression leads to the following minimization problem associated with Fredholm integral equation (1):

$$\underset{(a_{0m}, a_{\infty m})}{\text{minimize}} \int_0^1 (a_{0m}[P] g_0[x; P] + a_{\infty m}[P] g_\infty[x, P] - T_s[x])^2 dx, \tag{6}$$

where

$$g_0[x; P] = \int_0^1 \frac{G[x, x'; P]}{f_0[x']} dx', \tag{7}$$

and

$$g_\infty[x; P] = \int_0^1 \frac{G[x, x'; P]}{f_\infty[x']} dx'. \tag{8}$$

Minimizing the functional (6) leads to the following expressions for $a_{0m}[P]$ and $a_{\infty m}[P]$

$$a_{0m}[P] = \frac{G_0 G_{\infty^2} - G_{\infty 0} G_\infty}{G_0^2 G_{\infty^2} - G_\infty^2}, \tag{9}$$

and

$$a_{\infty m}[P] = \frac{G_0^2 G_\infty - G_{\infty 0} G_0}{G_0^2 G_{\infty^2} - G_\infty^2}, \tag{10}$$

where

$$G_0[P] = \int_0^1 g_0[x; P] T_s[x] dx,$$

$$G_\infty[P] = \int_0^1 g_\infty[x; P] T_s[x] dx,$$

$$G_0^2[P] = \int_0^1 (g_0[x; P])^2 dx,$$

$$G_{\infty^2}[P] = \int_0^1 (g_\infty[x; P])^2 dx,$$

$$G_{\infty 0}[P] = \int_0^1 g_0[x; P] g_\infty[x; P] dx. \tag{11}$$

The expressions for the coefficients $a_{0m}[P]$ and $a_{\infty m}[P]$ (Eqs. (9) and (10)), along with the integrals ((7), (8) and (11)), constitute an approximate explicit result for the unknown function $q[x; P]$ (Eq. (5)). In the next section, we compare results obtained through least squares regression, with both asymptotic

متن کامل مقاله

دریافت فوری ←

ISIArticles

مرجع مقالات تخصصی ایران

- ✓ امکان دانلود نسخه تمام متن مقالات انگلیسی
- ✓ امکان دانلود نسخه ترجمه شده مقالات
- ✓ پذیرش سفارش ترجمه تخصصی
- ✓ امکان جستجو در آرشیو جامعی از صدها موضوع و هزاران مقاله
- ✓ امکان دانلود رایگان ۲ صفحه اول هر مقاله
- ✓ امکان پرداخت اینترنتی با کلیه کارت های عضو شتاب
- ✓ دانلود فوری مقاله پس از پرداخت آنلاین
- ✓ پشتیبانی کامل خرید با بهره مندی از سیستم هوشمند رهگیری سفارشات