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# Logistic regression analysis for experimental determination of forming limit diagrams

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**Abstract**

The forming limit diagram (FLD) is probably the most common representation of sheet metal formability and can be defined as the locus of the principal planar strains where failure is most likely to occur. Experimental determination of the FLD consists in performing a set of formability tests on a sheet metal blank, where a regular grid has been previously etched. After each test, the deformation of the grid is measured and the relative strains computed. Strains observed closely at the fracture location are related to as 'failed' points, while strains observed on the sound areas of the specimens are labelled as 'safe' points. Starting from a set of experimental tests, the FLD should be empirically determined through a statistical analysis of collected data. In fact, statistical approaches (such as linear regression) are required to properly account for the internal randomness of failure occurrence. Linear regression, as well as most of the other empirical approaches in the scientific literature, takes into account only information related to the safe points.

This paper proposes a different approach, the logistic regression, for the empirical determination of FLDs. Logistic regression allows to directly derive the probability of an event (e.g. the failure) as a function of different predictor variables (both the principal planar strains). Therefore, by using logistic regression, the process designer can directly associate the failure probability to the scrapping costs, in order to economically evaluate a new sheet metal forming operation.

Logistic regression allows the determination of the FLD by including information concerning both safe and failed points.


*Keywords:* FLD; Forming limit band; Logistic regression; Sheet metal forming

## 1. Introduction

Although affected by many drawbacks, and although often obtained by laborious experiments, the Forming Limit Diagram (FLD), introduced by Keeler [1] and Goodwin [2], is probably the most widely used method for representing sheet metal formability and a very extensive literature has been produced on the subject in the last decades. The forming limit diagram represents the locus of failure points in the plane of principal strains $e_1-e_2$, where the symbols $e_1$ and $e_2$ will be used indifferently for both true and engineering major and minor strains, obtained under different strain paths (Fig. 1).

Experimentally, FLDs are determined by tests that follow approximately linear strain paths, such as the tensile test and the bulge test. However, if deformation paths are close to balanced biaxial stretching or to simple shear, ductile fracture due to void formation can be induced before the onset of localized necking [3–5]. Besides, in case of materials with low ductility, fracture often occurs without any obvious necking phenomenon also for other paths. Therefore, a more useful definition of FLD, which will be adopted in this paper, can be given as the locus of onset of *failure* [6], where failure means:

- necking, if necking occurs;
- fracture, if necking does not occur.

An FLD can be obtained only after etching a regular grid onto an undeformed sheet metal blank and by measuring the strains on the deformed grid after a formability test. More particularly, FLDs are determined by measuring

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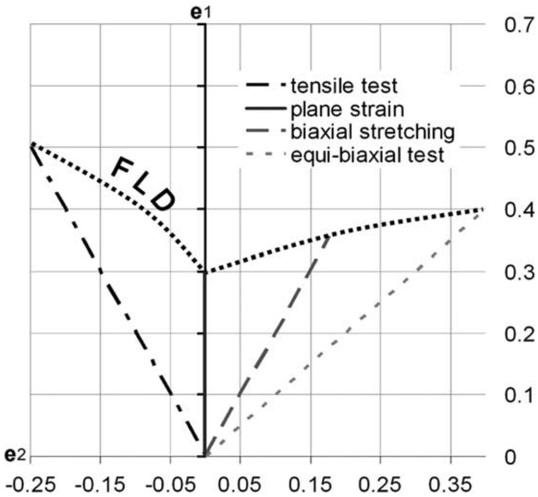


Fig. 1. Typical FLD diagram.

the deformation of the etched grid closely at the fracture location. Each 'failed' point will be plotted on the $e_1-e_2$ Cartesian plane and labelled accordingly. However, the deformed grid can be measured also on the safe areas of the deformed specimen, thus providing a set of 'safe' data points on the $e_1-e_2$ plane. An example of measured strain data is plotted in Fig. 2, where three kinds of experimental points ('safe', 'failed' and 'almost failed') can be observed. Material points are considered to be 'almost failed' if they are safe, but very close or surrounded by failed points. However, many authors do not make a difference between safe and almost failed points.

For industrial purposes, the knowledge of the *average* shape and position of FLD does not usually provide enough information. Due to uncertainty, the users are generally more interested in knowing the width of a forming limit band (FLB), i.e. a small region below which virtually no failure occurs [7]. An FLB is shown in Fig. 2: clearly, the position of the lower bound is technologically more relevant, since it identifies a fail-safe limit.

Starting from a set of experimental tests, the FLD can be empirically built using a statistical analysis of collected data. In fact, statistical approaches are required to properly account for the unavoidable randomness of the failure phenomenon. Linear regression is a very common tool for experimental analysis of data, when the response variable is continuous. If the response variable can assume only a small number of values (i.e. is dichotomous or ordinal), linear regression is unfit to correctly model the data and alternative techniques, such as logistic regression, should be used. In this case linear regression can be used to identify the functional relationship, which links one principal planar strain (acting as the response variable) to the other (acting as the predictor variable) for the observed failed points. Therefore, linear regression does not allow to take into account information related to safe points. It can provide an estimation of the FLB as the region where a given percentage of failure points (e.g. 90 or 99%) is predicted to lie within.

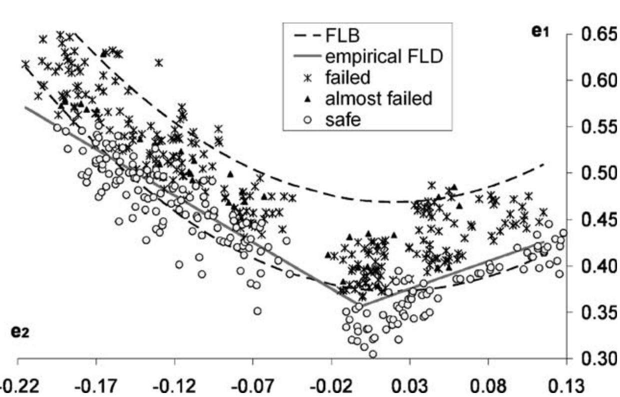


Fig. 2. FLD with safe; failed and almost failed points (AKDQ steel, sheet thickness 0.7 mm, as received [10]).

In cases where the variability of experimental failed points is large (as in Fig. 2) and the safe and unsafe regions are not clearly separated, finding a feasible process design might be difficult for the process engineer. Indeed, the best information for efficiently planning a sheet metal working process, would be the failure risk associated with each strain combination. When an FLD is known, the risk of a given solution can be roughly approximated by the minimum distance from the FLD or, alternatively, from the lower bound of the FLB.

In this paper the use of logistic regression is proposed as an effective statistical method, for determining an FLD. Logistic regression is a commonly used statistical technique (useful in many application in biostatistics, medical research, epidemiology, environmental science, data mining, industrial statistics) and is part of a category of statistical models called generalized linear models [8,9]. Logistic regression allows to predict a discrete outcome, from a set of variables that may be continuous, discrete, dichotomous, or a mix of any of these. Generally, the dependent or response variable is dichotomous, such as success/failure. Logistic regression, like linear regression, allows to estimate the relationship between some predictor variable (in this case the major and minor strains) and an outcome variable (failure/success). In logistic regression, however, one is interested in estimating the probability that the outcome variable assumes a certain value (rather than estimating the value itself). Therefore, with reference to FLD model building, logistic regression allows to directly estimate the probability of failure (or failure 'risk'), by considering both the failed and safe experimental points in the analysis.

In Section 2, a critical review of current experimental methods for estimation of forming limits is presented, together with a survey on the use of statistics in the scientific literature. In Section 3, the mathematical details of linear regression are given and discussed. In Section 4, the use of binary logistic regression is described and proposed as an effective way of generating a complete formability map in