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Fuzzy Sets and Systems 117 (2001) 13–25

FUZZY sets and systems

www.elsevier.com/locate/fss

Fuzzy dynamic programming approach to hybrid multiobjective multistage decision-making problems

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Received November 1997; received in revised form September 1998

Abstract

The purpose of this paper is to develop a new fuzzy dynamic programming approach for solving hybrid multiobjective multistage decision-making problems. We first present a methodology of fuzzy evaluation and fuzzy optimization for hybrid multiobjective systems, in which the qualitative and quantitative objectives are synthetically considered. The qualitative objectives are evaluated by decision-makers with linguistic variables and the quantitative objectives are converted into proper dimensionless indices. After getting the marginal evaluations for each objective, a new aggregation method based on the principle of fuzzy pattern recognition is developed to get a global evaluation for all objectives. With the global evaluation obtained, a fuzzy optimization process is performed. Then we present a dynamic optimization algorithm by incorporating the fuzzy optimization process with the conventional dynamic programming technique to solve hybrid multiobjective multistage decision-making problems. A characteristic feature of the approach proposed is that various objectives are synthetically considered by the fuzzy systematic technique instead of the frequently employed weighted average method. Finally, an illustrative example is also given to clarify the developed approach and to demonstrate its effectiveness.

Keywords: Fuzzy set; Multiobjective decision-making; Multistage decision-making; Fuzzy optimization; Dynamic programming

1. Introduction

Dynamic Programming (DP) is a powerful optimization apparatus for dealing with a large spectrum of complex problems involving sequential or multistage decision-making in many areas, e.g., control theory, pattern recognition, operations research, systems analysis, etc. Such problems occur and are relevant in virtually all human activities. There are many imprecise and uncertain factors due to man's inherent subjectivity and vagueness in the articulation of their opinions. For obvious reasons, the analysis of multistage decision-making problems by conventional DP is rather difficult under fuzzy environments. Assuming that Zadeh's fuzzy sets theory was an appropriate

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0165-0114/01/$ - see front matter
PII: S0165-0114(98)00423-0

way to deal with uncertainties and imprecision in real-world problems, DP was one of the earliest fundamental methodologies to which fuzzy sets theory was applied [4], leading to what might be called fuzzy dynamic programming (FDP). FDP has received wide attention in many research and application fields during the last ten years. Numerous contributions to FDP of both a foundational and an application character have appeared in the literature (cf. [2, 4, 8–13, 15–17, 22]). Excellent reviews of FDP appear in the literature [16, 17].

Recently, multiobjective dynamic programming (MODP), which relies heavily on the conventional DP technique, is developed as a technique for solving problems that involve various objectives (which are often conflicting) that possess the DP characteristics (refer [1, 5, 13, 19, 21]). Most methods proposed in the literature have a common character, i.e. they only deal with quantitative objectives. The essentials of the approach proposed in the literature are converting the multiobjective problem into a single-objective problem in some way, both crisp or fuzzy modeling, and then solving it by the conventional dynamic programming technique.

The purpose of this paper is to present a new effective and efficient fuzzy dynamic programming approach to hybrid multiobjective multistage decision-making problems with qualitative and quantitative objectives. Our approach is based fundamentally on the technique of fuzzy synthetic evaluation and fuzzy optimization for a multiobjective system and on the conventional principles of dynamic programming. The feature of our approach is that the fuzzy multiobjective optimization process is performed dynamically at each stage instead of converting the multiobjective problem into a single-objective problem.

The paper is structured as follows: In Section 2, the hybrid multiobjective multistage decision-making problem is formulated. Section 3 concerns the fuzzy synthetic evaluation and fuzzy optimization model for a hybrid multiobjective system. Section 4 designs an efficient and effective algorithm of fuzzy dynamic programming for solving hybrid multiobjective multistage decision-making problems. Section 5 gives a numerical example to demonstrate the effectiveness of the proposed approach and Section 6 concludes the paper.

2. Hybrid multiobjective multistage decision-making problems

In the general setting assumed here, we have a deterministic system whose dynamics are described by a state transition equation

$$s_{t+1} = f(s_t, x_t) \quad (t = 1, 2, \ldots, T), \tag{2.1}$$

where $s_t \in S_t$ is a (crisp) state variable taking its value in the set S_t of values permitted at stage (time) t, and $x_t \in X_t$ is a (crisp) decision variable taking its value in the set X_t of possible decisions at stage t; $S_t = \{\sigma_1^{(t)}, \sigma_2^{(t)}, \ldots, \sigma_{n_t}^{(t)}\}$ and $X_t = \{a_1^{(t)}, a_2^{(t)}, \ldots, a_{m_t}^{(t)}\}$ $(t = 1, 2, \ldots, T)$ are assumed to be finite throughout this paper.

The performance effects of the multistage decision-making process are evaluated by M hybrid objectives $O = \{O_1, O_2, \ldots, O_M\}$, where some of the objectives are qualitative and others are quantitative. We assume that each objective O_j $(j = 1, 2, \ldots, M)$ be decomposable, that is

$$y_j(\boldsymbol{x}) = \bigoplus_{t=1}^{T} y_j^{(t)}(x_t), \tag{2.2}$$

where $\boldsymbol{x} = (x_1, x_2, \ldots, x_T)$; $y_j^{(t)}(x_t)$ represents the performance result achieved by objective O_j when decision x_t is made at stage t $(t = 1, 2, \ldots, T)$ and $y_j(\boldsymbol{x}) = y_j(x_1, x_2, \ldots, x_T)$ represents the overall performance effect achieved by O_j when the decision sequence $\{x_1, x_2, \ldots, x_T\}$ is made during the multistage decision-making process. For simplicity, we present the performance results $y_j^{(t)}(x_t)$ in the cardinal form for the quantitative objectives and in the fuzzy linguistic variable form for the qualitative objectives, where the fuzzy linguistic variable values are in a finite set of natural evaluation words or phrase, such as {*Extremely Good*, *Very good*, *Good*, *Fair*, *Poor*, *Very Poor*, *Extremely Poor*}. Without loss of generality, we also suppose that the quantitative objectives are expected to maximize the total sum of their performance results achieved at each stage and the qualitative objectives are expected to optimize the whole synthetic effect of their performance results achieved at each stage. Thus, a *hybrid multiobjective multistage* (HMOMS) decision-making problem can be described as follows: