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Design of a fuzzy finite capacity queuing model based on the degree of customer satisfaction: Analysis and fuzzy optimization

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Abstract

Due to uncontrollable factors, the parameters in the queuing models may be uncertain and so in this paper we present the design of a fuzzy finite capacity queuing model in which the arrival pattern and the service pattern follow an exponential distribution under uncertain parameter. As for the design of the system, the criterion of optimization proposed is aimed at the optimum selection of the number of servers, with the goal of providing a high degree of satisfaction to clients when they join the system. This optimization process will be solved through Markov chains with fuzzy state. The validity of the procedure proposed to incorporate fuzzy data in this queuing system is confirmed by a fuzzy simulation experiment of the differential equations, which govern the behaviour of the model, by the Bontempi's Qua. Si. III algorithm. The extension of queuing decision models to fuzzy environments enables the decision maker to obtain more informative results and wider knowledge on the behaviour of the system, since the results obtained in the fuzzy queuing model are fuzzy subsets containing the whole initial information; that is why the finite capacity queuing models with uncertain data can have a broader range of applications.

Keywords: Fuzzy system model; Finite capacity queuing model; Markov Chain with fuzzy states; Fuzzy simulation

1. Introduction

Basic queuing models assume that the capacity of a facility has an unlimited number of units available. Arrivals are then controlled by a Poisson process which is to be independent of the number of units present within the system for a given time interval. However, queuing models sometimes have a finite queue or a finite system capacity so that the number of customers in the facilities is not allowed to be greater than the fixed capacity (denoted by N). Any unit arriving when the queue is "full" is prevented from entering the facilities and becomes a missing unit so that the average arrival rate in the system becomes zero at those moments. The most usual physical interpretation for this model is that only one finite waiting room, which accommodates just N units in the system, exists.

The waiting lines models with finite capacity have several applications in many practical situations and are analysed, among others, by Smith [23], Balsamo et al. [1], Bocharov and Viskova [2], Jain [14], Gupta and Sikdar [13] and

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Thomas [27]. Moreover, several studies have been published on finite capacity queuing models and their optimization, such as the papers by Wang and Ke [28], Ke and Wang [17], Takagi et al. [26] and Ziya et al. [36].

Within the context of traditional queuing theory, the time that elapses between the arrivals and the service duration is determined by probability distributions. However, in many practical situations, the statistical information is obtained subjectively, so that the arrival pattern and the service pattern are more adequately described by linguistic terms, such as fast, slow or moderate, rather than by probability distributions. For this reason, fuzzy queuing models offer more appropriate information about the system than classical models commonly used. If the classic finite capacity queuing model is extended to the fuzzy model, it can have a greater number of applications and may be easier to use in practice.

Although fuzzy queuing decision problems are more practical than the conventional ones, and finite capacity queuing models with fuzzy parameters are more realistic, few studies on this topic have been published: for example, on the basis of possibility theory, Buckley [4] discussed elementary multiple-server queuing systems with finite or infinite capacity, in which the arrivals and the departures follow a possibility pattern and Buckley et al. [5] extended the previous results [4] to fuzzy queuing decision problems. Chen [7] proposed a parametric programming method to construct the membership function of the fuzzy objective function for a queuing decision problem in which the cost coefficients and the arrival rate are fuzzy numbers; Zhang and Phillis [34] determined the policy for optimal client appointment for a queuing system in parallel with two heterogeneous servers using fuzzy control; Pardo and de la Fuente [20] optimize a priority-discipline queuing model using fuzzy set theory and in [21] they have calculated the optimal selection of the service rate for a finite input source fuzzy queuing system, and de la Fuente and Pardo [12] analyse a finite capacity queuing model with uncertain parameters.

In this paper, we investigate a finite capacity queuing model, s servers and orderly waiting procedure so that on the basis of the classical model $M/M/s/N$ [25,30] with a Poisson input process and service durations in accordance with an exponential distribution, uncertain data are added to some of its initial starting hypotheses so obtaining the fuzzy queuing model $\tilde{M}/\tilde{M}/s/N$. Once we have developed the fuzzy finite capacity queuing model, we have carried out the optimization of the system by finding the adequate number of servers, s. To this end, we have considered providing a high degree of satisfaction to the customers when they join the system as the aim, based on which the expected satisfaction of a customer or "degree of customer satisfaction on joining the system" has been defined. This expected satisfaction is calculated through the following fuzzy states of the system: "short length of queue in the system" (providing a high degree of satisfaction), "average length of queue in the system" (providing an average degree of satisfaction) and "long length of queue in the system" (providing a zero degree of satisfaction), and it is studied along with the concepts of probability of a fuzzy event and Markov chains with fuzzy states.

The optimization—fixing the minimum level of acceptable satisfaction and calculating the number of servers that keep this satisfaction—can also be solved by other methods, such as the multicriteria optimization: one criterion is to maximize the customers' satisfaction and, the other one is to minimize the number of servers, for example by using Pareto-optimal solutions (e.g. [29]), even though we present the methodology to apply Markovian decision process with fuzzy states for the optimization of queuing models with uncertain data.

The handling of the finite capacity queuing model with uncertain data follows the works carried out by Prade [22], Li and Lee [18], Kao et al. [15], Negi and Lee [19] and Ke and Lin [16], who analysed fuzzy queuing models through Zadeh's extension principle [33], and the works presented by Pardo and de la Fuente [20,21] and de la Fuente and Pardo [12] which have calculated the optimization and design of several fuzzy queuing decision problems.

The queuing system with finite capacity holds that the system of differential equations that governs the model's behaviour is made up of a finite number of equations. This feature will enable us to carry out, with an example proposed to clarify and show the theoretical results, a fuzzy simulation experiment of the differential equations by the algorithm Qua. Si. III of Bontempi [3]. The simulation will provide us with the development over time of the fuzzy probabilities $\tilde{p}_i(t)$, their convergence on the fuzzy steady state probabilities $\tilde{\pi}_i$ and, moreover, we shall be able to compare the results of the fuzzy simulation with the results obtained analytically by the application of Zadeh's extension principle. In this way the validity of the procedure proposed to incorporate fuzzy data in queuing models is confirmed by the fuzzy simulation.

2. Fuzzy finite capacity queuing model, $\tilde{M}/\tilde{M}/s/N$

Finite capacity classic queuing models consider that the system's average input rate becomes zero when the capacity of the facilities is complete. Since the distributions of the times between arrivals and the service times are exponential,