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Numerical solution of dynamic optimization problems with flexible inequality constraints by iterative dynamic programming

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Abstract

A solution strategy for optimizing the dynamic systems with flexible inequality constraints is proposed. To apply fuzzy inference in solution, the flexible portion in the problem is treated as fuzzy constraints. After functional values are bounded in a region, the objective function of this problem can also be fuzzified easily. When the problem is formulated as a fuzzy dynamic optimization problem, the iterative dynamic programming integrated with fuzzy inference is adopted to find the solution. Two examples are employed, demonstrating the facility of the proposed algorithm. © 2002 Elsevier Science B.V. All rights reserved.

Keywords: Fuzzy mathematical programming; Fuzzy constraint satisfaction; Iterative dynamic programming

1. Introduction

Since 1960, the optimization of transient behavior for chemical processes has received significant attention [5,10,15,4]. All these investigations share a common assumption that each portion in the problem, including objective function (goal), system dynamics, and constraints are all definite in general. However, real-world situations are not so rigid. For example, the operator may moderately relax the limitation on the use of resource or cost to exchange the improvement of quality. Such flexibility in operation largely depends on the operator's subjective considerations. In addition, engineers also need to make decisions under the circumstance with uncertain factors. However, the current dynamic optimization techniques becomes futile as the problem contains flexible portion. The aim of this paper is to find a solution strategy for dynamic systems subjected to flexible inequality constraints.

The fuzzy dynamic optimization problems with single or multiple objectives under deterministic or fuzzy environment has been discussed in literature [14]. In [14], both the dynamic equations and the control policy were discretized, the so-called complete parametrization [4], the problem was then solved by the goal programming technique. Despite making the problem easily port on existent softwares, complete parametrizations not only largely expand dimension for properly approximating highly nonlinear dynamics, but also cause convergence difficulties for the system with multimodal nature.

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In this study, the fuzzy set concept, initialized by [18], will be used to grade the degree of acceptability for a flexible constraint. Then, the transformation method, proposed by [16,17] for solving linear static optimization problems with flexible constraints, will be adopted to grade the degree of satisfaction for the objective function. By way of such a fuzzification on the problem, the fuzzy dynamic programming, developed by [1], will be applied for further computation. Obviously, when dynamic programming (DP) is used to solve the dynamic optimization problem (DOP), the drastic expansion in problem's dimension, the well-known *curse of dimensionality*, will become unavoidable. To avoid this difficulty, Luss [10] proposed the use of coarse but accessible state grids and region contracting to reduce dimension expansion, and the iterative computation to promote reliability of the solution. The above modification of DP is called the iterative dynamic programming (IDP).

In the rest of this paper, Section 2 introduces the formulation of the problems and the fuzzification of objective function and flexible constraints. Section 3 briefly reviews fuzzy optimization. Section 4 presents the solution algorithms, the fuzzy dynamic programming and the iterative dynamic programming. Section 5 provides numerical illustrations of the proposed algorithm. Conclusions are finally made in Section 6.

2. Problem formulation

Consider the following dynamic optimization problem:

$$\max_{\mathbf{u}(t)\in\tilde{\Omega}} J[\mathbf{x}(t_f)], \tag{1}$$

where $J[\mathbf{x}(t_f)]$ denotes the objective function to be minimized and $\tilde{\Omega}$ represents the feasible space of control policy, in which all control policies $\mathbf{u}(t)$ satisfy system dynamics and flexible inequality constraints, i.e. $\tilde{\Omega} \equiv \{\mathbf{u}(t) \mid \dot{\mathbf{x}}(t) = \mathbf{f}(\mathbf{x}(t), \mathbf{u}(t)), \mathbf{x}(0) = \mathbf{x}_0; \mathbf{g}(\mathbf{x}(t), \mathbf{u}(t)) \leqslant \tilde{\mathbf{b}}; \underline{\mathbf{u}}(t) \leqslant \mathbf{u}(t) \leqslant \bar{\mathbf{u}}(t)\}$. Here, $\mathbf{x}(t) \in \mathbb{R}^n$ and $\mathbf{u}(t) \in \mathbb{R}^m$ are state and control vectors, respectively, $\mathbf{f}(\mathbf{x}(t), \mathbf{u}(t)) \in \mathbb{R}^n$ and $\mathbf{g}(\mathbf{x}(t), \mathbf{u}(t)) \in \mathbb{R}^K$. Elements in $\tilde{\mathbf{b}} = [\tilde{b}_1, \ldots, \tilde{b}_K]^{\mathrm{T}}$ are the flexible boundaries for the constraints.

Eq. (1) seems to be a stochastic or probabilistic programming problem. If so, the flexible portions $\tilde{b}_k$'s will behave randomly during simulation. In practice, the sources of random variables depend on the nature and the type of the problem. For example, in the design of aircraft the actual loads acting on the plane depends on the atmospheric conditions, which cannot be predicted precisely in advance [12]. In Eq. (1), however, we assume that the operator does not randomly change the values of $\tilde{b}_k$'s, but changes on his preference.

2.1. Fuzzification of flexible constraints

As previously stated, the system's behavior is affected by a set of flexible inequality constraints, $g_k(\mathbf{x}(t), \mathbf{u}(t)) \leqslant \tilde{b}_k,\ k = 1, \ldots, K$. Here, the so-called flexible inequality constraint means that when the reasonable limiting value b_k and the acceptable maximal tolerance p_k can be preliminarily defined, all those values that are smaller than $b_k + p_k$ can be regarded as satisfying g_k. Therein, all values less than b_k are thoroughly satisfied. For values in between b_k and $b_k + p_k$, however, the extent satisfying g_k decreases with an increase of its value. To quantitatively demonstrate such an linguistic character, a fuzzy set $\mathscr{C}_k$ with $\mu_{\mathscr{C}_k}(\mathbf{x}(t), \mathbf{u}(t))$ denoting the degree of acceptability for the inequality constraint $g_k(\mathbf{x}(t), \mathbf{u}(t))$, is defined.

$$\mu_{\mathscr{C}_k}(\mathbf{x}(t), \mathbf{u}(t)) = \begin{cases} 1 & \text{if } g_k(\mathbf{x}(t), \mathbf{u}(t)) < b_k, \\ \mathbb{F}_{\mathscr{C}_k}(\mathbf{x}(t), \mathbf{u}(t), b_k, p_k) & \text{if } b_k \leqslant g_k(\mathbf{x}(t), \mathbf{u}(t)) \leqslant b_k + p_k, \\ 0 & \text{if } g_k(\mathbf{x}(t), \mathbf{u}(t)) > b_k + p_k, \end{cases} \tag{2}$$

where $\mathbb{F}$ denotes any monotonic decreasing function, and is used to describe the membership value for $\mu_{\mathscr{C}_k}$. The general criteria for defining $\mathbb{F}_{\mathscr{C}_k}$ are somehow subjective. Two types of monotonic decreasing membership functions commonly used are listed in the following (also shown in Figs. 1 and 2):

$$\mathbb{F}_{\mathscr{C}_k}(\mathbf{x}(t), \mathbf{u}(t), b_k, p_k) = \begin{cases} \frac{b_k + p_k - g_k(\mathbf{x},\mathbf{u})}{p_k} & \text{for linear type,} \\ \frac{1 - \exp[-\alpha(b_k + p_k - g_k(\mathbf{x},\mathbf{u}))/p_k]}{1 - \exp[-\alpha]} & \text{for exponential type,} \end{cases} \tag{3}$$