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Adaptive differential dynamic programming for multiobjective optimal control ☆

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Abstract

An efficient numerical solution scheme entitled adaptive differential dynamic programming is developed in this paper for multiobjective optimal control problems with a general separable structure. For a multiobjective control problem with a general separable structure, the "optimal" weighting coefficients for various performance indices are time-varying as the system evolves along any noninferior trajectory. Recognizing this prominent feature in multiobjective control, the proposed adaptive differential dynamic programming methodology combines a search process to identify an optimal time-varying weighting sequence with the solution concept in the conventional differential dynamic programming. Convergence of the proposed adaptive differential dynamic programming methodology is addressed.

Keywords: Multiobjective optimal control; Dynamic programming; Multiobjective dynamic programming; Differential dynamic programming; Adaptive differential dynamic programming

1. Introduction

Many real-world control problems are characterized by their multiple performance measures that are often noncommensurable and conflict with each other. During the past three decades the development of multiobjective control (Zadeh, 1963; Salukvadze, 1979; Toivonen & Mäkilä, 1989; Li, 1990; Khargonekar & Rotea, 1991; De Nicolao & Locatelli, 1992; Li, 1993a,b; Carvalho & Ferreira, 1995) has grown by leaps and bounds. Based on the principle of optimality, multiobjective dynamic programming (Brown & Strauch, 1965; Yu & Seiford, 1981; Li & Haimes, 1987,1989) is a powerful solution methodology in solving multiobjective control problems.

The curse of dimensionality in dynamic programming prevents its direct adoption in many real-world large-scale control problems. Reducing the dimensionality in dynamic programming (Larson & Korsak, 1970; Morin & Esogbue, 1974; Bertsekas & Castañon, 1989; Haurie & L'Ecuyer, 1986; Johnson, Stedinger, & Shoemaker, 1993; Mayne, 1966; Jacobson & Mayne, 1970; Yakowitz & Rutherford, 1984; Liao & Shoemaker, 1991; Luus, 1998) has been a challenging research task in front of the control and optimization community for many years. The curse of dimensionality of dynamic programming is further aggravated in situations with a vector-valued objective function. There is, however, no report in the literature on efficient numerical algorithms for multiobjective optimal control problems with a general separable structure.

We consider in this paper the following general class of multiobjective optimal control problems:

(P)

$$\min_{U} \quad J(X,U) = (J_1(X,U), J_2(X,U), \ldots, J_k(X,U))' \qquad (1)$$

$$\text{s.t.} \quad x_{t+1} = f_t(x_t,u_t),\ t = 0,\ldots,T-1,\ x_0 \text{ given}, \qquad (2)$$

where $U = (u_0', \ldots, u_{T-1}')'$, $X = (x_0', x_1', \ldots, x_T')'$, $x_t \in R^n$, $t = 0,\ldots,T$, and $u_t \in R^p$, $t = 0,\ldots,T-1$. In problem (P),

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all performance indices $J_i(X,U)$, $i=1,\dots,k$, are assumed to be backward separable. More specifically, there exist functions $\phi_t^{[i]}: R^n \times R^p \times R \to R$, $t=0,\dots,T-1$, and $\phi_T^{[i]}: R^n \to R$, $i=1,2,\dots,k$, such that each $J_i(X,U)$ can be expressed by a backward nested form:

$$y_T^{[i]} = \phi_T^{[i]}(x_T), \quad i=1,\dots,k,$$
$$y_t^{[i]} = \phi_t^{[i]}(x_t,u_t,y_{t+1}^{[i]}), \quad i=1,\dots,k, \ \ t=T-1,\dots,0, \quad (3)$$
$$J_i(X,U) = y_0^{[i]}, \quad i=1,\dots,k,$$

where $\partial y_t^{[i]}/\partial y_{t+1}^{[i]} > 0$ for all $i=1,\dots,k$, $t=0,1,\dots,T-1$. (This assumption ensures the satisfaction of "the monotonicity condition".) Eq. (3) can be rewritten in a compact form:

$$y_T = \phi_T(x_T),$$
$$y_t = \phi_t(x_t,u_t,y_{t+1}), \quad t=T-1,\dots,0, \quad (4)$$
$$J(X,U) = y_0,$$

where

$$y_t = (y_t^{[1]},\dots,y_t^{[k]})', \ t=0,\dots,T,$$
$$\phi_T(x_T) = (\phi_T^{[1]}(x_T),\dots,\phi_T^{[k]}(x_T))',$$
$$\phi_t(x_t,u_t,y_{t+1}) = (\phi_t^{[1]}(x_t,u_t,y_{t+1}^{[1]}),\dots, \phi_t^{[k]}(x_t,u_t,y_{t+1}^{[k]}))',$$
$$t=0,\dots,T-1,$$

and

$$J(X,U) = (J_1(X,U),\dots,J_k(X,U))'.$$

We assume in this paper that all functions have continuous second-order derivatives.

Problem formulation in (P) covers all types of multiobjective control problems with a separable structure in the sense of multiobjective dynamic programming. Different performance indices, J_i, $i=1,\dots,k$, in (P) may take different function forms and the composite operations in a performance index J_i may vary from stage to stage. General separable performance indices that are not stagewise additive often result from optimization problems in areas such as minimax control, reliability optimization, multi-reservoir system, chemical engineering process, and mathematical programming, see, for example, Tauxe, Inamn, and Mades (1979), Mine and Fukushima (1979), Li (1995) and Luus (1997). Scalar-valued dynamic programming with a general separable performance index is studied in Nemhauser (1966) and Furukawa and Iwamoto (1976). Multiobjective dynamic programming with general separable performance indices is studied in Yu and Seiford (1981) and Li and Haimes (1987). There are some special features in this general class of multiobjective control problems. When some functions of J_i's are of nonlinear stagewise forms, an attempt of scalarization to convert problem (P) into a scalar-valued optimal control problem will lead to a formulation that is nonseparable in the sense of dynamic programming. In addition, the approach using state augmentation will also fail when some performance indices are backward separable, but not forward separable.

Multiobjective optimal control problem (P) with a general separable structure can be solved by using the envelope approach (Li & Haimes, 1987), a multiobjective dynamic programming method. When an analytical solution form is not obtainable, the numerical implementation of the envelope approach with discretization in computation is not mathematically tractable, or even infeasible when the problem dimension is large. The limitation of the envelope approach comes about from discretization, thus the curse of dimensionality of dynamic programming. This dimensionality problem becomes more serious in multiobjective dynamic programming than in the single-objective dynamic programming, since instead of recording the scalar cost-to-go and its associated optimal control, the vector-valued cost-to-go and its associated set of noninferior solutions have to be recorded for each grid of the state at each stage.

In this paper, we aim at developing an efficient numerical solution method, entitled adaptive differential dynamic programming, for problem (P). This new solution method extends the idea, concept, and the solution scheme of differential dynamic programming (Mayne, 1966; Jacobson & Mayne, 1970; Yakowitz & Rutherford, 1984; Liao & Shoemaker, 1991) to multiobjective dynamic programming. The dimensionality problem in multiobjective dynamic programming will be overcome in the proposed new method since no discretization is involved.

The presentation of this paper is as follows. Some basic properties of problem (P) will be discussed in the next section. Adaptive differential dynamic programming will be developed in Section 3 for (P). The convergence analysis of adaptive differential dynamic programming will be carried out in Section 4. Numerical implementation of adaptive differential dynamic programming is shown in Section 5. The paper concludes in Section 6 with some concluding remarks.

2. Preliminaries

Solving multiobjective control problem (P) entails finding the set or a representative subset of noninferior solutions. A solution (X^*,U^*) of (P) is said to be *noninferior* (Chankong & Haimes, 1983) if there exists no other feasible (X,U) such that $J_i(X,U) \leqslant J_i(X^*,U^*)$ for all i and at least one strict inequality holds. We define

$$EZ = \{U \mid U \text{ is a noninferior solution to } (P)\}.$$

The principle of optimality is the cornerstone upon which dynamic programming is built. The principle of optimality established in multiobjective cases (Li & Haimes, 1987) enables an adoption of multiobjective dynamic programming for solving separable multiobjective control problems presented in (P).