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Dynamic programming in a heuristically confined state space: a stochastic resource-constrained project scheduling application

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Abstract

The resource-constrained project scheduling problem (RCPSP) is a significant challenge in highly regulated industries, such as pharmaceuticals and agrochemicals, where a large number of candidate new products must undergo a set of tests for certification. We propose a novel way of addressing the uncertainties in the RCPSP including the uncertainties in task durations and costs, as well as uncertainties in the results of tasks (success or failure) by using a discrete time Markov chain, which enables us to model probabilistic correlation of the uncertain parameters. The resulting stochastic optimization problem can be solved by using dynamic programming, but the computational load renders this impractical. Instead, we develop a new way to combine heuristic solutions through dynamic programming in the state space that the heuristics generate. The proposed approach is tested on a simplified version of RCPSP that has a fairly complicated stochastic nature, with 1,214,693,756 possible parameter realizations (scenarios), and involves five projects and 17 tasks. As a result, an on-line policy is obtained, which can use the information states in on-line decision making and improve the heuristics rather than a fixed solution obtained by the previous mathematical programming (MILP) problem formulations.

Keywords: Stochastic dynamic programming; Markov chain; Project scheduling; Combinatorial optimization; Heuristics

1. Introduction

A challenge in highly regulated industries, such as pharmaceuticals and agrochemicals, is the process of selecting, developing, and efficiently manufacturing new products that emerge from the discovery phase. Candidate products must undergo a set of tests related to safety, efficacy, and environmental impact, to obtain certification. The problem of scheduling these tasks and associated analysis can be considered as a generalization of the well-known job shop scheduling problem. The case in which all the problem data have known values belongs to the NP-hard class of combinatorial problems (Blazewicz, Lenstra, & Kan, 1983). In general, task success or failure is uncertain and the time value of project reward varies, which adds more complexity to the scheduling problem. In a specialized R&D pipeline management problem, the time value of project reward decreases as the time to introduction of the product increases due to incoming competitive products and fixed patent periods. Hence a company has to manage its various resources, manpower, lab space, capital, pilot facilities, etc. to ensure its best return on its new product pipeline, with the added complication that the outcome of tasks is uncertain. Besides the uncertainty about the success of the task, there are several additional uncertain parameters in real problems, such as uncertainties in task duration and resource (cost) requirement.

The project scheduling problem with unlimited resource (Schmidt & Grossmann, 1996) was introduced to the process systems engineering area using a mathematical programming (MILP) based solution approach. In the case of unlimited resource, the overall objective function (net present value) of the problem can be separated into the individual objective functions of each project since one project does not influence the others. There has been significant progress in solution methods (Blau et al., 2000; Jain & Grossmann, 1999; Maravelias & Grossmann, 2001; Rogers, Gupta, & Maranas, 2002) for the problem with resource constraints as well as uncertainty in the task outcome. However, previous

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solution methods for resource-constrained project scheduling problem (RCPSP) have considered only a subset of the potential uncertainties and have been based on mathematical programming techniques. Even though the mathematical programming approach can account for uncertainties of the problem via scenario generation, the approach is limited to a fairly small number of scenarios due to the exponential increase in the computational load. Limitations in the mathematical programming approach lie not only in the computational tractability but also in the awkwardness in capturing richer representations of uncertainty. Notable exceptions are Subramanian, Pekny, and Reklaitis (2000, 2001, 2003), where a broader set of uncertainties in the problem are addressed within a simulation and optimization (SIMOPT) framework. The SIMOPT framework developed in Subramanian et al. (2000, 2001, 2003) achieved substantial improvement in combining stochastic simulation and optimization by taking a discrete-event dynamic system's view of the RCPSP. However, outer iteration process of the SIMOPT where constraints are added to the MILP to steer it away from decisions that gave poor outcomes in simulation cannot does not fully and rigorously account for the way information and outcomes can influence the decisions.

In this study, we address the uncertainties in the RCPSP using a discrete time Markov chain, which enables us to model correlations among the uncertain parameters. For example, the probability of success of a future task may not be independent of the outcomes of current or previous tasks. Furthermore, a novel solution method, dynamic programming in a heuristically confined state space developed and illustrated in Choi, Lee, and Realff (2002a, 2003) and Choi, Realff, and Lee (2002b), is tailored to the problem to obtain high quality solutions. The proposed approach is focused on solving the RCPSP as a multi-stage on-line decision making problem. Finally, the proposed approach is demonstrated by effectively solving a fairly complex stochastic RCPSP that can have up to 1.2 billion different outcomes depending on realization of the uncertainty.

2. Problem description: stochastic RCPSP

We consider a simplified version of RCPSP with M projects, each of which consists of m_i tasks, for $i = 1, \dots, M$. There are N resources (laboratories), a specific resource has to be used to perform each task. In the example formulation studied in this paper, the resources are represented as laboratories (Lab.). Several problem parameters of a task, the result (success or failure), the duration, and the cost, are uncertain. A detailed description of the uncertainty model is given in Section 2.1. A time-varying reward function is given for each project to represent the decreasing value of the project with time. The reward function (Eq. (1)) is characterized by three parameters: 'stiffness parameter', α, 'project deadline indicator', PD, and 'final value', β.

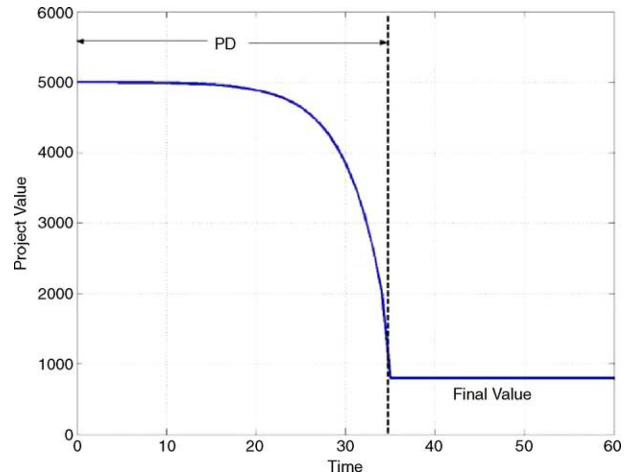

Fig. 1. Decreasing reward function.

$$\begin{aligned} R(0) &= R_0, && \text{at} \quad k = 0 \\ R(k) &= R_0 - \mathrm{e}^{\alpha k}, && \text{for} \quad 0 < k \leq \text{PD} \\ R(k) &= \beta, && \text{for} \quad k > \text{PD} \end{aligned} \tag{1}$$

Fig. 1 shows the reward function with $R_0 = 5000$, $\alpha = 0.235$ and PD = 34.

2.1. Uncertain parameter modeling: Markov chain and conditional probability

RCPSPs with diverse representations of uncertain parameters have been addressed in the literature (Blau et al., 2000; Jain & Grossmann, 1999; Maravelias & Grossmann, 2001; Rogers et al., 2002; Schmidt & Grossmann, 1996; Subramanian et al., 2000, 2001, 2003). However, it appears that probabilistic correlation among the uncertain parameters in the RCPSP has not been addressed previously. Our problem representation is based on the premise that the result, duration and cost of adjacent tasks in a project are correlated. For example, if a current task takes longer to complete, then the duration of the next task also tends to be longer. The assumption is particulary appropriate for the drug development pipeline management problem because a candidate (drug) has to pass similar types of tests with varying number of patients. In general, the correlation can exist between any two tasks in a project and can be modeled by introducing corresponding transition probability. However, in this paper we assume the probabilistic correlation between two adjacent tasks only for simplification. The probabilistic correlation among uncertain parameters can be modeled with discrete time Markov chains. The nth task of a project i has r_{ni} realizations and each realization consists of the values of the result, duration, and cost of the task from a discrete set as shown in Fig. 2. For example, 'F, D_{11i}, C_{11i}' (the first realization set of the task 1 in Fig. 2) represents failure of the task with D_{11i} duration and C_{11i}