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Using dynamic programming with adaptive grid scheme for optimal control problems in economics

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Abstract

The study of the solutions of dynamic models with optimizing agents has often been limited by a lack of available analytical techniques to explicitly find the global solution paths. On the other hand, the application of numerical techniques such as dynamic programming to find the solution in interesting regions of the state was restricted by the use of fixed grid size techniques. Following Grüne (Numer. Math. 75 (3) (1997) 319; University of Bayreuth, submitted, 2003), in this paper an adaptive grid scheme is used for finding the global solutions of discrete time Hamilton–Jacobi–Bellman equations. Local error estimates are established and an adapting iteration for the discretization of the state space is developed. The advantage of the use of adaptive grid scheme is demonstrated by computing the solutions of one- and two-dimensional economic models which exhibit steep curvature, complicated dynamics due to multiple equilibria, thresholds (Skiba sets) separating domains of attraction and periodic solutions. We consider deterministic and stochastic model variants. The studied examples are from economic growth, investment theory, environmental and resource economics.

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1. Introduction

In recent times the lack of closed form solutions of dynamic models with optimizing agents has led to the use of computational methods to solve those models. Closed

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form solutions are available for special cases such as the linear-quadratic control problem, see Ljungqvist and Sargent (2001, Chapter 4) and growth models with log-utility. While those special models suffice for the analytical study of a variety of economic problems in more general cases they are not sufficient for a robust analysis of interesting economic problems. For the more general cases numerical methods have been proposed in the literature. A detailed discussion of a variety of numerical methods and a comparison with our proposed method is provided in Section 3 below.

Our paper is concerned with a family of continuous and discrete time dynamic models with optimizing agents whose solutions are most conveniently approached by the method of dynamic programming. Dynamic programming provides the value function and the control variable in feedback form. Yet, the application of numerical methods such as dynamic programming to find the global dynamics in interesting regions of the state space were restricted by the use of fixed grid size techniques. Following Grüne (1997) in this paper an adaptive grid scheme is used for finding global solutions of models with dynamic optimization. Like Santos and Vigo-Aguiar (1998), we use numerical value function iteration but we employ local error estimates based on a flexible grid scheme. Since those numerical methods provide us with approximate solutions only, it is essential to have accuracy estimates for the numerical methods employed.

We consider discounted continuous time and discrete time optimal control problems and flexible grid scheme based on local error estimates. The advantage of the use of flexible grid scheme is demonstrated by computing the value function and the control variable in feedback form of one and two dimensional economic models. In order to study the accuracy of our numerical methods when applied to economic models, we first want to test our algorithm by studying a basic growth model of Brock and Mirman (1972) type for which the exact solution is known. This allows us to judge the accuracy of our numerical method and to explore whether the flexible grid scheme is accurately capturing important solution properties, for example, steep curvature of the value function. Our algorithm will be applied to a deterministic as well as to a stochastic version of the Brock and Mirman (1972) growth model.

Moreover in the economic literature there exist more complicated dynamic models with optimizing agents which have been a challenge to commonly used numerical techniques. Those models exhibit more complicated dynamics due to the existence of multiple steady state equilibria,[1] thresholds (Skiba-sets) separating domains of attraction and periodic solutions as attractors. Examples of such models can be found in the literature on economic growth and development.[2] Multiple steady states and thresholds can also arise in the dynamic decision problem of the firm, for example due to relative

[1] If there are local attractors among the equilibria some authors characterize them as indeterminant equilibria. For a survey of models with indeterminacy, see Benhabib and Farmer (1999).

[2] In the latter type of models a convex–concave production function arises which leads to thresholds separating paths to low per capita income (poor) countries and high per capita income (rich) countries, see Skiba (1978) and Azariadis and Drazen (1990). In more recent growth models of the Lucas or Romer type multiple steady state growth paths may arise due to externalities and complementarities of inputs, or monopolistic competition, see for example, Benhabib et al. (1994) and Benhabib and Perli (1994).