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Applications of fixed point technique in solving certain dynamic programming and variational inequalities☆

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Abstract

In this paper, we introduce the concepts of compatible maps of type (T)/ type (I) and weakly compatible maps of type (T)/ type (I) for hybrid functions to produce common fixed point theorems for set-valued functions. Our results extend some known results for point-valued and set-valued functions. As applications of fixed point technique, the existence and uniqueness of common solutions for certain class of the functional equations in dynamic programming and variational inequalities arises in two point obstacle problem are discussed.

MSC: 54H10; 47H10

Keywords: Upper semicontinuous function; $(\varepsilon, \gamma, \Phi)(p, q)$ contraction; Compatible functions; δ-compatibility; Weakly compatible functions; Weakly compatible functions of type (*T*) and type (*I*)

1. Introduction

In this paper, we extend the concept of compatible functions of type (T)/ type (I) from point-valued functions to hybrid functions, that is, for point-valued and set-valued functions, and introduce the concept of weakly compatibly of type (T)/ type (I) for hybrid functions.

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In the sequel, we prove some fixed point theorems for hybrid functions without appealing to continuity of functions. As a tool we have used the concept of (weakly) compatible functions of type (I) and generalized Meir–Keeler contraction called $(\varepsilon, \gamma, \Phi)(p, q)$ contraction. Our results extend the results of Chang [6], generalized results by Jachymski [8], Kang and Rhoades [12] and Jungck and Rhoades [10,11]. Our results can also be viewed as byproduct generalizations of theorems for point-valued functions.

2. Basic concepts and definitions

Let (X, d) be a metric space and $B(X)$ the set of all nonempty bounded subsets of X. If $A, B \in B(X)$, then $\delta(A, B) = \sup\{d(a, b) : a \in A \text{ and } b \in B\}$, and $D(A, B) = \inf\{d(a, b) : a \in A \text{ and } b \in B\}$. Further, if $A = \{x\}$, then $D(\{x\}, B) = \inf\{d(x, b) : b \in B\}$ for $x \in X$. It is evidently that $0 \leqslant \delta(A, B) \leqslant \delta(A, C) + \delta(C, B)$ and $\delta(A, B) = 0$ iff $A = B = \{x\}$ for $A, B, C \in B(X)$ (see Jungck and Rhoades [10,11]). If $a \in X$, we write $\delta(a, B)$ for $\delta(\{a\}, B)$.

Definition 2.1. Let (X, d) be a metric space and let Φ be the collection of functions $\phi : [0, \infty) \to [0, \infty)$ which are upper semicontinuous from the right, nondecreasing and satisfying the condition:

$$\lim_{s \to t^+} \sup \phi(s) < t \quad \text{and} \quad 0 < \phi(t) < t \quad \text{for all } t < 0.$$

Let $S, T : X \to B(X)$. Then S and T are $(\varepsilon, \gamma, \Phi)(p, q)$ contractions relative to maps $I, J : X \to X$ iff $\cup S(X) \subseteq J(X), \cup T(X) \subseteq I(X)$, and there exist functions $p, q : X \times X \to [0, \infty), \gamma : (0, \infty) \to (0, \infty)$ such that $\gamma(\varepsilon) > \varepsilon \; \forall \; \varepsilon > 0, a \in [0, 1]$ and for $x, y \in X$:

$$(*) \quad 0 < \varepsilon \leqslant ap(x, y) + (1 - a)q(x, y) < \gamma(\varepsilon) \Rightarrow \delta(Sx, Ty) < \varepsilon,$$

$$p(x, y) = \max\{\phi_1(d(Ix, Jy)), \phi_2(\delta(Sx, Ix)), \phi_3(\delta(Ty, Jy)), \phi_4(\tfrac{1}{2}[D(Sx, Jy) + D(Ix, Ty)])\} \text{ and}$$

$$q(x, y) = \max\{\phi_5(d^2(Ix, Jy)), \phi_6(\delta^2(Sx, Ix)), \phi_7(D(Sx, Jy)D(Ix, Ty)), \phi_8(\max((D(Ix, Ty)D(Sx, Jy), \delta^2(Jy, Ty)))\}^{1/2}$$

$\phi_i \in \Phi (i = 1, 2, 3, 4, 5, 6, 7, 8)$. It may be observed that

(i) if $p(x, y) = \lambda(x, y) = \max\{\phi_1 d(Ix, Jy), \phi_2(\frac{1}{2}[D(Sx, Jy) + D(Ix, Ty)])\}$,

$$q(x, y) = \mu(x, y) = \max\{\phi_3(d^2(Ix, Jy)), \phi_4(D(Sx, Jy)D(Ix, Ty)), \phi_5(\max[D(Ix, Ty)D(Sx, Jy), \delta^2(Jy, Ty)])\}^{1/2},$$

then we refer to $(\varepsilon, \gamma, \Phi)(p, q)$ contractions as (λ, μ) contractions;

(ii) if $p(x, y) = \lambda(x, y) = \max\{\phi_1(d(Ix, Jy)), \phi_2(\frac{1}{2}[D(Sx, Jy) + D(Ix, Ty)])\}$ and $a = 1$, then we refer to $(\varepsilon, \gamma, \Phi)(p, q)$ contractions as (λ) contractions;