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Analysis of input-to-state stability for discrete time nonlinear systems via dynamic programming☆

Shoudong Huang[a], Matthew R. James[b,*], Dragan Nešić[c], Peter M. Dower[c]

[a]Centre of Excellence for Autonomous Systems, Faculty of Engineering, The University of Technology, Sydney, Australia
[b]Department of Engineering, Australian National University, Canberra, ACT 0200, Australia
[c]Department of Electrical and Electronic Engineering, The University of Melbourne, Parkville, 3010, Victoria, Australia

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Abstract

The input-to-state stability (ISS) property for systems with disturbances has received considerable attention over the past decade or so, with many applications and characterizations reported in the literature. The main purpose of this paper is to present analysis results for ISS that utilize dynamic programming techniques to characterize minimal ISS gains and transient bounds. These characterizations naturally lead to computable necessary and sufficient conditions for ISS. Our results make a connection between ISS and optimization problems in nonlinear dissipative systems theory (including L_2-gain analysis and nonlinear H_∞ theory). As such, the results presented address an obvious gap in the literature.
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1. Introduction

Among the many stability properties for systems with disturbances, the input-to-state stability (ISS) property proposed by Sontag (1989) deserves special attention. Indeed, ISS is fully compatible with Lyapunov stability theory (Sontag & Wang, 1995) while its other equivalent characterizations relate it to robust stability, dissipativity and input–output stability theory (Sontag, 2000; Sontag & Wang, 1996). The ISS property has found its main application in the ISS small gain theorem that was first proved by Jiang, Teel, and Praly (1994). Several different versions of the ISS small gain theorem that use different (equivalent) characterizations of the ISS property and their various applications to nonlinear controller design can be found in Jiang and Mareels (1997); Jiang, Mareels, and Wang (1996); Teel (1996) and references defined therein.

The ISS property and the ISS small gain theorems naturally lead to the concept of nonlinear disturbance gain functions or simply "nonlinear gains". In this context, obtaining sharp estimates for the nonlinear gains is an important issue. Indeed, the better the nonlinear gain estimate that we can obtain, the larger the class of systems to which the ISS small gain results can be applied. Currently, the main tool for estimating the nonlinear gains are the so-called ISS Lyapunov functions that typically produce rather conservative estimates (over bounds) for the ISS nonlinear gains.

It is the main purpose of this paper to present several results that provide a constructive framework based on dynamic programming for obtaining *minimum ISS nonlinear gains*. These results are related to optimization based methods in nonlinear dissipative systems theory, such as L_2-gain analysis and nonlinear H_∞ theory (see, Helton & James, 1999 and references defined therein), as well as recently developed optimization based L_∞ methods (see, Fialho & Georgiou, 1999; Huang & James, 2003 and references defined therein). Needless to say, the optimization approach that we take in this paper can inflict a heavy (and sometimes infeasible) computational burden on

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* Corresponding author. Tel.: +61 2 6125 4889; fax: +61 2 6125 0506.
E-mail addresses: sdhuang@eng.uts.edu.au (S. Huang), Matthew.James@anu.edu.au (M.R. James), d.nesic@ee.mu.oz.au (D. Nešić), p.dower@ee.mu.oz.au (P.M. Dower).

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the user. This is a reflection of the intrinsic complexity of the problem that we are trying to solve. We present results only for discrete-time nonlinear systems since many calculations and technical details are in this way simplified.

The paper is organized as follows. In Section 2 we present several equivalent definitions of the ISS property and state a result from the literature that motivates our definitions and results. A fundamental dynamic programming equation that we need to state our main results is given in Section 3. Sections 4, 5 and 6 contain results on minimum nonlinear gains for different equivalent definitions of the ISS property. Two related ISS properties are analysed in Section 7 using the techniques of Sections 5 and 6. Several illustrative examples are presented in Section 8 and the paper is closed with conclusions in Section 9.

2. Preliminaries

Sets of real numbers, integers and nonnegative integers are denoted, respectively, as $\mathbf{R}$, $\mathbf{Z}$ and $\mathbf{Z}_+$. A function $\gamma : [0,\infty) \to [0,\infty)$ is of class $\bar{\mathscr{K}}$ if it is nondecreasing, satisfies $\gamma(0) = 0$ and is right continuous at 0. A function $\beta : [0,\infty) \times [0,\infty) \to [0,\infty)$ is of class $\bar{\mathscr{K}}\bar{\mathscr{L}}$ if for each fixed $t \geqslant 0$, $\beta(\cdot, t)$ is of class $\bar{\mathscr{K}}$ and for each fixed $s \geqslant 0$, $\lim_{t\to+\infty}\beta(s,t) = 0$. Denote $l_\infty = \{u : \mathbf{Z}_+ \to \mathbf{R}^m : \|u\|_\infty = \sup_{k\in\mathbf{Z}_+} |u_k| < \infty\}$ where $|\cdot|$ is the Euclidean norm.

Consider the following dynamical system

$$x_{k+1} = f(x_k, u_k), \tag{1}$$

where $x_k \in \mathbf{R}^n$, $u_k \in \mathbf{R}^m$, and $f : \mathbf{R}^n \times \mathbf{R}^m \to \mathbf{R}^n$ is continuous and satisfies $f(0,0) = 0$. For any $x_0 \in \mathbf{R}^n$ and any input $u : \mathbf{Z}_+ \to \mathbf{R}^m$, we denote by $x(\cdot, x_0, u)$ the solution of (1) with initial state x_0 and input u.

The following definitions are taken from ISS related literature. It was shown in Jiang and Wang (2001) that these definitions of ISS are qualitatively equivalent. However, the gains in different definitions are not the same and since we are interested in minimum disturbance gains for different characterizations, we find it useful to introduce different notation for each of the different characterizations. In all the definitions below we assume that $\gamma \in \bar{\mathscr{K}}$ and $\beta \in \bar{\mathscr{K}}\bar{\mathscr{L}}$.

Definition 1 (*Input-to-state stability with + formulation*). System (1) is ISS_+ (with (β,γ)) if

$$|x(k, x_0, u)| \leqslant \beta(|x_0|, k) + \gamma(\|u\|_\infty) \tag{2}$$

for all $x_0 \in \mathbf{R}^n$, all $u \in l_\infty$ and all $k \in \mathbf{Z}_+$.

Definition 2 (*Asymptotic gain property*). System (1) is AG (with gain γ) if

$$\limsup_{k\to+\infty} |x(k, x_0, u)| \leqslant \gamma(\|u\|_\infty) \tag{3}$$

for all $x_0 \in \mathbf{R}^n$ and all $u \in l_\infty$.

Remark 3. Using arguments as in Lemma II.1 of Sontag and Wang (1996), we can show that the above definition is equivalent to the following: for all $x_0 \in \mathbf{R}^n$ and all $u \in l_\infty$,

$$\limsup_{k\to+\infty} |x(k, x_0, u)| \leqslant \gamma\left(\limsup_{k\to+\infty} |u_k|\right), \tag{4}$$

which is the definition of asymptotic gain property in Jiang and Wang (2001).

Definition 4 (*Zero global asymptotic stability property*). System (1) is 0-GAS (with β) if the state trajectories with $u \equiv 0$ satisfy

$$|x(k, x_0, 0)| \leqslant \beta(|x_0|, k) \tag{5}$$

for all $x_0 \in \mathbf{R}^n$ and all $k \in \mathbf{Z}_+$.

Definition 5 (*Input-to-state stability with asymptotic gain formulation*). System (1) is ISS_{AG} (with (β,γ)) if it is AG (with gain γ) and 0-GAS (with β).

Remark 6. The above definition is motivated by the result proved in Sontag and Wang (1996) which shows for continuous-time systems that $ISS_+ \Leftrightarrow AG + 0\text{-}GAS$. A similar result for discrete-time systems was proved in Gao and Lin (2000), Jiang and Wang (2001). This result is restated below in Theorem 9 for convenience.

Definition 7 (*Input-to-state stability with* max *formulation*). System (1) is $\mathrm{ISS}_{\max}$ (with (β,γ)) if

$$|x(k, x_0, u)| \leqslant \max\{\beta(|x_0|, k), \gamma(\|u\|_\infty)\} \tag{6}$$

for all $x_0 \in \mathbf{R}^n$, all $u \in l_\infty$ and all $k \in \mathbf{Z}_+$.

Remark 8. It is more common in the literature to use the classes of functions $\mathscr{K}$ and $\mathscr{K}\mathscr{L}$ when defining ISS and related properties. A function $\gamma : [0,\infty) \to [0,\infty)$ is of class $\mathscr{K}$ if it is continuous, strictly increasing and $\gamma(0) = 0$. A continuous function $\beta : [0,\infty) \times [0,\infty) \to [0,\infty)$ is of class $\mathscr{K}\mathscr{L}$ if for each fixed $t \geqslant 0$, $\beta(\cdot, t)$ is of class $\mathscr{K}$ and for each fixed $s \geqslant 0$ $\beta(s, \cdot)$ decreases to zero.

It is not hard to see that the stability definitions that we use are qualitatively equivalent to the stability definitions when the classes of functions $\bar{\mathscr{K}}$ and $\bar{\mathscr{K}}\bar{\mathscr{L}}$ are replaced, respectively, by $\mathscr{K}$ and $\mathscr{K}\mathscr{L}$. This follows from the following three facts: (i) $\mathscr{K} \subset \bar{\mathscr{K}}$ and $\mathscr{K}\mathscr{L} \subset \bar{\mathscr{K}}\bar{\mathscr{L}}$; (ii) given any $\gamma \in \bar{\mathscr{K}}$, there exists $\gamma_1 \in \mathscr{K}$ such that $\gamma(s) \leqslant \gamma_1(s)$, $\forall s \geqslant 0$; (iii) given any $\beta \in \bar{\mathscr{K}}\bar{\mathscr{L}}$, there exists $\beta_1 \in \mathscr{K}\mathscr{L}$ such that $\beta(s,k) \leqslant \beta_1(s,k)$, $\forall s \geqslant 0$, $\forall k \in \mathbf{Z}_+$. Consequently, most results that were proved in the literature for classes of functions $\mathscr{K}$ and $\mathscr{K}\mathscr{L}$ are still true when stated with function classes $\bar{\mathscr{K}}$ and $\bar{\mathscr{K}}\bar{\mathscr{L}}$.

Finally, we note that our relaxed function class definitions are necessitated by the fact that the minimal ISS gain for some systems can be of class $\bar{\mathscr{K}} \backslash \mathscr{K}$, as is demonstrated in Section 8.1, Example 1.

The following theorem has been proved in the context of function classes $\mathscr{K}$ and $\mathscr{K}\mathscr{L}$ for continuous-time systems in Sontag and Wang (1996) and for discrete-time systems in Gao and Lin (2000), Jiang and Wang (2001). However, this result remains valid for function classes $\bar{\mathscr{K}}$ and $\bar{\mathscr{K}}\bar{\mathscr{L}}$.

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