A Dual Dynamic Programming For Multidimensional Parabolic Optimal Control Problems

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In the paper the optimal control problems governed by parabolic equations are considered. We apply a new dual dynamic programming approach to derive sufficient optimality conditions for such problems. The idea is to move all the notions from a state space to a dual space and to obtain a new verification theorem providing the conditions which should be satisfied by a solution of the dual partial differential equation of dynamic programming. We also give sufficient optimality conditions for the existence of an optimal dual feedback control and some approximation of the problem considered which seems to be very useful from the practical point of view.

Keywords: Dual Dynamic Programming; Dual Feedback Control; Parabolic Equation; Optimal Control Problem; Sufficient Optimality Conditions; Verification Theorem

1. Introduction

Consider the following optimal control problem (P):

minimize
$$J(x, u) = \int_{[0,T] \times \Omega} L(t, z, x(t, z), u(t, z)) dt dz$$

 $+ \int_{\Omega} l(x(T, z)) dz$

subject to

$$x_t(t,z) + \Delta_z x(t,z)$$

$$= f(t, z, x(t, z), u(t, z)) \text{ a.e. on } [0, T] \times \Omega \quad (1)$$

$$x(0,z) = \varphi(0,z) \text{ on } \Omega$$
 (2)

$$x(t,z) = \psi(t,z) \text{ on } [0,T] \times \partial \Omega$$
 (3)

$$u(t,z) \in U$$
 a. e. on $[0,T] \times \Omega$ (4)

where Ω is a given subset of \mathbb{R}^n which is bounded with Lipschitz boundary and U is a given nonempty set in $R^m; L, f: [0, T] \times \Omega \times R \times R^m \to R, l: R \to R$ and $\varphi: \mathbb{R}^{n+1} \to \mathbb{R}$ are given functions; $x: [0, T] \times$ $\Omega \to R, x \in W^{2,2}(\Omega)$ and $u: [0,T] \times \Omega \to R^m$ is a Lebesgue measurable function. We assume that for each s in R, the functions $(t, z, u) \rightarrow L(t, z, s, u)$, $(t, z, u) \rightarrow f(t, z, s, u)$ are $(L \times B)$ -measurable, where $L \times B$ is the σ -algebra of subsets of $[0, T] \times \Omega \times R^m$ generated by products of Lebesgue measurable subsets of $[0, T] \times \Omega$ and Borel subsets of \mathbb{R}^m , and for each $(t, z, u) \in [0, T] \times \Omega \times \mathbb{R}^m$, the functions $s \to L(t, z, s, u), s \to f(t, z, s, u)$ are continuous. We call a pair (x(t,z), u(t,z)) to be admissible if it satisfies (1)–(4) and L(t, z, x(t, z), u(t, z)) is summable; then the corresponding trajectory x(t, z) is said to be admissible.

The aim of the paper is to present sufficient optimality conditions for problem (P) in terms of

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dynamic programming conditions directly. In the literature, there is not work in which problem (P) is studied directly by a dynamic programming method. The only results known to the authors (see e.g. [1]– [11], [12], [13] and references therein) treat problem (P) as an abstract problem with an abstract evolution equation (1) and later derive from abstract Hamilton-Jacobi equations the suitable sufficient optimality conditions for problem (P). We propose almost a direct method to study (P) by a dual dynamic programming approach following the method described in [14] for one dimensional case and in [9] for multidimensional case. We move all notions of a dynamic programming to a dual space (the space of multipliers) and then develop a dual dynamic approach together with a dual Hamilton-Jacobi equation and as a consequence sufficient optimality conditions for (P). We also define an optimal dual feedback control in the terms of which we formulate sufficient conditions for optimality. Such an approach allows us to weak significantly the assumptions on the data. An approximate minimum in terms of the dual dynamic programming is also investigated.

2. A Dual Dynamic Programming

In this section we describe an intuition of a dual dynamic approach to optimal control problems governed by parabolic equations. Let us recall what does a dynamic programming mean? We have an initial condition $(t_0, x_0(t_0, z)), z \in \Omega$ for which we assume that we have an optimal solution $(\overline{x}, \overline{u})$. Then by necessary optimality conditions there exists a conjugate function $\overline{p}(t,z) = (\overline{y}^0, \overline{y}(t,z))$ on $[0,T] \times \Omega$ being a solution to the corresponding adjoint system (see e.g. [5], [11]). The element $p = (y^0, y)$ plays a role of multipliers from the classical Lagrange problem with constraints (with multiplier y^0 staying by the functional and *y* corresponding to the constraint). If we perturb (t_0, x_0) then, assuming that the optimal solution exists for each perturbed problem, we also have a conjugate function corresponding to it. Therefore making perturbations of our initial conditions we obtain two sets of functions: optimal trajectories \overline{x} and corresponding to them conjugate functions \overline{p} . The graphs of optimal trajectories cover some set in a state space (t, z, x), say a set X (in the classical calculus of variation it is named the field of extremals), and the graphs of conjugate functions cover some set in a conjugate space (t, z, p), say a set P (in classical mechanics it is named the space of momentums). In the classical dynamic programming

approach we explore the state space (t, z, x), i.e. the set X (see e.g. [1]) but in the dual dynamic programming approach we explore the conjugate space (the dual space) (t, z, p), i.e. the set P (see [14] for one dimensional case and [9] for multidimensional case). It is worth to note that although in elliptic control optimization problems we have not possibilities to perturb that problems, the dual dynamic programming is still possible to be applied (see [10]). It is natural that if we want to explore the dual space (t, z, p) then we need a mapping between the set P and the set $X: P \ni (t, z, p) \to (t, z, \tilde{x}(t, z, p)) \in X$ to have a possibility to formulate, at the end of some consideration in P, any conditions for optimality in our original problem as well as on an optimal solution \overline{x} . Of course, such a mapping should have the property that for each admissible trajectory x(t, z) lying in X we must have a function p(t, z) lying in P such that $x(t, z) = \tilde{x}(t, z, p(t, z))$. Hence, we conduct all our investigations in a dual space (t, z, p), i.e. most of our notions concerning the dynamic programming are defined in the dual space including a dynamic programming equation which becames now a dual dynamic programming equation.

Therefore let $P \subset \mathbb{R}^{n+3}$ be a set of the variables $(t, z, p) = (t, z, y^0, y), (t, z) \in [0, T] \times \Omega, y^0 \leq 0, y \in \mathbb{R}$, and let $\overline{c} = (c^0, c) \in \mathbb{R}^2$ be fixed. The constant \overline{c} is introduced because of the practical purpose only, i.e. in order to make easier the calculations of some relation stated below for concrete problems (see section An Example). We adopt the convention that $(t, z, \overline{c}p) = (t, z, c^0y^0, cy)$ for $(t, z, p) \in P$. Let $\tilde{x} : P \to R$ be such a function that for each admissible trajectory x(t, z) there exists a function $p(t, z) = (y^0, y(t, z)), p \in W^{2,2}([0, T] \times \Omega), (t, z, p(t, z)) \in P$ such that

$$x(t,z) = \tilde{x}(t,z,p(t,z)) \text{ for } (t,z) \in [0,T] \times \Omega.$$
(5)

Now, let us introduce an auxiliary C^2 function $V(t, z, p) : P \to R$ such that for $(t, z, p) \in P$, $(t, z, \overline{c}p) \in P$ the following condition is satisfied

$$V(t, z, \overline{c}p) = c^0 y^0 V_{y^0}(t, z, \overline{c}p) + cy V_y(t, z, \overline{c}p)$$

= $\overline{c}p V_p(t, z, \overline{c}p).$ (6)

The condition (6) is the generalization of a tranversality condition known in classical mechanics as the orthogonality of a momentum to the front of a wave. Similarly as in the classical dynamic programming define at $(t, p(\cdot))$, where $p(z) = (y^0, y(z))$ is any

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