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Developing coal pillar stability chart using logistic regression

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ABSTRACT

Logistic regression was utilised to calculate the probability that a particular pillar of a given geometry (width to height ratio) and a known stress condition (strength to stress ratio) will be stable. The stable-failure boundary was also determined. The logistic regression was also used to calculate and draw isoprobability contours. These contours represent the probability of stability of coal pillars based on the probability function for each stability class and are a valuable design tool in quantifying the instability probability of coal pillars.

1. Introduction

Pillars in underground coal mines are mainly designed to ensure the protection of roadways and entries. Pillars stability is consequently the most important factor that must be guaranteed through the entire life of mine that can be years or even decades long. Pillar stability can be analysed by a number of methods that are generally based on the ratio between the pillar strength and pillar load that is expressed in Factor of Safety (*FoS*).

The strength characteristics of coal pillars has been studied by many researchers and the subject has been well discussed in the literature, for examples [1–5]. A number of fairly intensive new developments relating to coal pillar strength estimation methods have also been carried out.

Two coal pillar strength approaches based on a tentative empirical failure criterion for coal seams were proposed [6]. The first, progressive failure type approach, gave incorrect estimates of pillar strength for low width-height ratios while the second pillar strength approach performed satisfactorily for both slender and fiat pillars. The performance of these new equations has been compared with some of the more popular strength formulas and tested against 16 failed and 27 stable pillar case studies.

A new coal pillar strength equation was developed and tested, along with existing equations, against 23 failed and 20 stable case studies [7]. The results revealed that in situ strength is more affected by depth of cover than indicated by laboratory tests, and a new safety factor based on depth and width/height ratio was proposed.

Alternative methods of coal pillar strength estimation using numerical modelling with strain softening constitutive behaviour of coal were also provided. The numerical models implemented in the researches were three dimensional finite difference [8] and finite element [9] methods. They might give decent results and better than the previous pillar strength estimation method.

Furthermore, it was found that the *FoS* calculated using deterministic approach had some intrinsic limitations in handling uncertainties in material properties, non-regular geometries and different mining operations [10]. A probabilistic expression for *FoS* was then suggested, which provided a confidence interval to express the reliability of coal pillar stability.

In this paper, a statistical approach based on real stable and failed cases of coal pillars was suggested. As pillar stability was only defined as stable or failed, logistic regression was utilised because the method is suitable for categorical dependent variables.

2. Logistic regression analysis

2.1. Logistic regression model

Logistic regression is a statistical modelling technique where the dependent variable (Y) has only two possible values and it is a useful tool for analysing data that includes categorical response variables, such as yes/no or live/die or stable/failed, as compared to the regression of numerical values. Logistic regression does not model the dependent variable directly but it is rather based on the probabilities associated with the values of Y. For simplicity, and because it is the case most commonly encountered in practice, Y can be coded as 1 in the case positive outcome or

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success (i.e. yes or live or stable) and coded as 0 in the case of negative outcome (i.e. no or die or failed). If there is a collection of p independent variables denoted by the vector $\boldsymbol{x}'=(x_1, x_2,\ldots, x_p)$, the hypothetical population proportion of cases for which $Y=1$ is defined as [11]

$$P(Y=1/x)=p(x) \tag{1}$$

and the specific form of the logistic regression model is

$$p(x)=\frac{e^{\beta_0+\beta_1x_1+\beta_2x_2+\ldots+\beta_px_p}}{1+e^{\beta_0+\beta_1x_1+\beta_2x_2+\ldots+\beta_px_p}} \tag{2}$$

where β_0, $\beta_1,\ldots$, β_p are the logistic regression model parameters.

For theoretical mathematical reasons, logistic regression is based on a linear model for the natural logarithm of the odds in favour of $Y=1$, which are simply the ratio of the proportions for the two possible outcomes [12], written as

$$Odds=\frac{\pi(x)}{1-\pi(x)} \tag{3}$$

and the general form of the logistic regression model can now be written as:

$$\ln\left[\frac{\pi(x)}{1-\pi(x)}\right]=\beta_0+\beta_1x_1+\beta_2x_2+\ldots+\beta_px_p \tag{4}$$

The log-odds, as defined above is also known as the logit transformation of $\pi(x)$ and the related analysis is sometimes known as logit analysis.

2.2. Logistic regression model fitting

If there is a sample of n independent observations (x_i,y_i), $i=1, 2,\ldots, n$, the model fitting requires the estimates of the vector $\beta'=(\beta_0, \beta_1, \beta_2,\ldots, \beta_p)$ and the general method of estimation is called maximum likelihood. In a very general sense, the method of maximum likelihood yields values for the unknown parameters which maximise the probability of obtaining the observed data [11]. In order to utilise the method, a likelihood function must be first constructed, in the form of

$$l(\boldsymbol{\beta})=\prod_{i=1}^{n}\pi(x_i)^{Y_i}[1-\pi(x_i)]^{1-Y_i} \tag{5}$$

It is mathematically easier to work with the log of Eq. (5), and this expression, the log likelihood, is defined as

$$L(\boldsymbol{\beta})=\ln[l(\boldsymbol{\beta})]=\sum_{i=1}^{n}\{Y_i\ln[\pi(x_i)]+(1-Y_i)\ln[1-\pi(x_i)]\} \tag{6}$$

To find the values of $\boldsymbol{\beta}$ that maximise $L(\boldsymbol{\beta})$, Eq. (6) is differentiated with respect to β_0, β_1, $\beta_2,\ldots$, β_p and resulting equations, known as the likelihood equations, are set to zero, as follows:

$$\sum_{i=1}^{n}[Y_i-\pi(x_i)]=0 \tag{7}$$

and

$$\sum_{i=1}^{n}x_{ij}[Y_i-\pi(x_i)]=0 \tag{8}$$

for $j=1, 2,\ldots, p$.

In linear regression, the likelihood equations are linear in the unknown parameters and easily solved. In logistic regression, the expressions in Eq. (7) and Eq. (8) are nonlinear in β_0, β_1, $\beta_2,\ldots$, β_p and special methods are required for their solutions. These methods are iterative in nature and have been programmed into available logistic regression software. A general discussion of the methods used by most programs may be seen in [13] where it is shown that, in particular, the solution to Eqs. (7) and (8) may be obtained using an iterative weighted least squares procedure [11].

3. Coal pillar case histories

3.1. Coal pillar stability data

Coal pillar case histories have been compiled by [14], as listed in Table 1. The pillars are grouped into stable (denoted by 1) and failed (denoted by 0) pillars for the purpose of logistic regression analysis.

As pillar stability is theoretically governed by the width to height ratio and strength to stress ratios, the logistic regression analysis conducted in this paper was based on these two ratios. The strengths of pillars in Table 1 were estimated using the equations proposed by [15] as follows:

$$S=0.27\sigma_ch^{-0.36}+\left(\frac{H}{250}+1\right)\left(\frac{w}{h}-1\right)(\text{MPa}) \tag{9}$$

where S is pillar strength in MPa, σ_c is uniaxial compressive strength of coal in MPa, H is pillar location depth in m, w is pillar width in m, and h is pillar height in m.

Pillar stresses in Table 1 were estimated simply by using the tributary area theory where the unit weight of coal bearing strata overburden was taken as 0.025 MN/m^3 as follows:

$$P=0.025H\frac{(w+B)^2}{w^2}(\text{MPa}) \tag{10}$$

where P is pillar stress in MPa and B is roadway width in m.

Theoretically, a pillar with S/P ratio, which is basically the *FoS*, greater than 1.0 would be stable. However, as can be seen in Table 1, the stable pillars have S/P ratios in the range of 0.84–5.55, whereas the failed pillars have S/P ratios in the range of 0.78–1.37. It is then obvious, that there is still a possibility of failure for pillar with a S/P ratio greater than 1.0.

3.2. Logistic regression model from coal pillar stability data

The logistic regression model from pillar stability data was developed with the independent variables of w/h $(=x_1)$ and S/P $(=x_2)$ and the dependent variable (Y) is stability that coded as 1 for stable pillar and 0 for failed pillar. The resulted model is

$$P\left(\text{pillar is stable}\,|\left(\frac{W}{h}\right),\left(\frac{S}{P}\right)\right)=\frac{\exp\left[-13.146+2.774\left(\frac{w}{h}\right)+5.668\left(\frac{S}{P}\right)\right]}{1+\exp\left[-13.146+2.774\left(\frac{w}{h}\right)+5.668\left(\frac{S}{P}\right)\right]} \tag{11}$$

The probability of stability for each pillar can be calculated and compared to the actual stability, as depicted in Table 2. The table reveals that the logistic regression model estimated that the probability of a stable pillars is stable is in the range of 29.53%–100%, whereas that of a failed pillars is in the range of 0.19%–87.04%. Table 2 also shows that the proposed logistic regression model predict low probability of stability (i.e. 29.53%) for one stable pillar and on the other hand predict high probability of stability (i.e. 87.04%) for one failed pillar. These two cases are indicated with (*) in Table 2. It can also be observed that based on the estimated probability of stability, the accuracy of the proposed logistic regression model is acceptable, as given in Table 3 that shows the comparison between actual and predicted stability.

4. Coal pillar stability chart

4.1. Stable-failure boundary line

The logistic regression determines the orientation of the stable-failure boundary line that can be used to separate the different stability categories. Determination of the position of this boundary was conducted following the similar approach to that was used in determining the stable-failure and failure-caving boundaries for block