ELSEVIER

Fuzzy Sets and Systems 109 (2000) 35–53

FUZZY sets and systems

www.elsevier.com/locate/fss

Evolutionary algorithm solution to fuzzy problems: Fuzzy linear programming

James J. Buckley[a,*], Thomas Feuring[b,1]

[a]Department of Mathematics, University of Alabama at Birmingham, Birmingham, AL 35294, USA
[b]Institut für Informatik, Westfälische Wilhelms-Universität Münster, Einsteinstraße 62, 48149 Münster, Germany

Received July 1997; received in revised form January 1998

Abstract

In this paper we wish to find solutions to the fully fuzzified linear program where all the parameters and variables are fuzzy numbers. We first change the problem of maximizing a fuzzy number, the value of the objective function, into a multi-objective fuzzy linear programming problem. We prove that fuzzy flexible programming can be used to explore the whole undominated set to the multi-objective fuzzy linear program. An evolutionary algorithm is designed to solve the fuzzy flexible program and we apply this program to two applications to generate good solutions.

Keywords: Mathematical programming; Fuzzy linear programming; Evolutionary algorithms

1. Introduction

In this paper we wish to construct solutions to the fully fuzzified linear program (written FFLP)

$$\max \quad \overline{Z} = \overline{C}_1\overline{X}_1 + \cdots + \overline{C}_n\overline{X}_n \tag{1}$$

$$\text{s.t.} \quad \overline{A}_{i1}\overline{X}_1 + \cdots + \overline{A}_{in}\overline{X}_n \leqslant \overline{B}_i, \quad 1 \leqslant i \leqslant m, \tag{2}$$

$$\overline{X}_i \geqslant 0 \quad \text{for all } i, \tag{3}$$

where the $\overline{C}_i$, $\overline{A}_{ij}$ and $\overline{B}_i$ are all triangular fuzzy numbers and the $\overline{X}_i$ are also triangular fuzzy numbers. We will code the FFLP as

$$\max \quad \overline{Z} = \overline{C}\,\overline{X} \tag{4}$$

$$\text{s.t.} \quad \overline{\boldsymbol{A}}\overline{X} \leqslant \overline{B}, \quad \overline{X} \geqslant 0, \tag{5}$$

where $\overline{C} = (\overline{C}_1, \ldots, \overline{C}_n)$, $\overline{X}^t = (\overline{X}_1, \ldots, \overline{X}_n)$, $\overline{B}^t = (\overline{B}_1, \ldots, \overline{B}_m)$ and $\overline{\boldsymbol{A}} = [\overline{A}_{ij}]$ a $m \times n$ matrix of fuzzy numbers. Before we discuss the contents of the paper, let us introduce the basic notation to be employed. We place a bar over a capital letter to denote a fuzzy subset of the real numbers. So $\overline{A}$, $\overline{B}$, $\overline{C}$, $\overline{X}$, etc., are all fuzzy subsets of the real numbers. We write $\overline{A}(x)$, a number in $[0, 1]$, for the membership function of $\overline{A}$ evaluated as x. An α-cut of $\overline{A}$, written as $\overline{A}[\alpha]$, is defined as $\{x \mid \overline{A}(x) \geqslant \alpha\}$, for $0 < \alpha \leqslant 1$. We separately

* Corresponding author. Tel.: +1-205-934-2154; fax: +1-205-934-9025.

E-mail address: buckley@math.uab.edu (J.J. Buckley)

[1] This work was supported with a grant from the German Academic Exchange Service (DAAD) based on the Hochschulsonderprogramm III of Bund and Länder while staying at the University of Alabama at Birmingham.

0165-0114/00/$ – see front matter
PII: S0165-0114(98)00022-0

specify $\overline{A}[0]$ as the closure of the union of all the $\overline{A}[\alpha]$, $0 < \alpha \leqslant 1$.

A triangular fuzzy number $\overline{N}$ is defined by three numbers $a_1 < a_2 < a_3$ where the graph of $\overline{N}(x)$ is a triangle with base on the interval $[a_1, a_3]$ and vertex at $x = a_2$. We specify $\overline{N}$ as $(a_1/a_2/a_3)$. The degenerate cases of $a_1 = a_2 < a_3$ and $a_1 < a_2 = a_3$, where we get half of a triangular fuzzy number, are allowed and we still call them triangular fuzzy numbers. A triangular-shaped fuzzy number $\overline{M}$ is partially defined by three number $a_1 < a_2 < a_3$ where: (1) the graph of $\overline{M}(x)$ is continuous and monotonically increasing from zero to one on $[a_1, a_2]$; (2) $\overline{M}(a_2) = 1$; and (3) the graph of $\overline{M}(x)$ is continuous and monotonically decreasing from one to zero on $[a_2, a_3]$. We write $\overline{M} \geqslant 0$ if $a_1 \geqslant 0$, $\overline{M} > 0$ if $a_1 > 0$, $\overline{M} \leqslant 0$, if $a_3 \leqslant 0$ and $\overline{M} < 0$ for $a_3 < 0$. We will use standard fuzzy arithmetic, from the extension principle, to evaluate sums, products, etc. of fuzzy numbers. The α-cut of a fuzzy number is always a closed and bounded interval.

To properly define the FFLP we must do two things: (1) define what we mean by $\max \overline{Z}$, or finding the "maximum" of a fuzzy number; and (2) explain what is meant by $\overline{E}_i \leqslant \overline{B}_i$, where $\overline{E}_i = \overline{A}_{i1}\overline{X}_1 + \cdots + \overline{A}_{in}\overline{X}_n$. How we are going to handle the problem of "maximizing" a fuzzy number is the topic of the next section and in Section 3 we discuss methods of evaluating fuzzy inequalities.

Searching for the optimal $\overline{X}$ to the FFLP is a very difficult problem. Let us assume that the meaning of "optimal" has been defined and the FFLP has an optimal solution. Then we do not know of an algorithm that will compute the exact value of the optimal $\overline{X}$. In the next section we turn the search for the optimal $\overline{X}$ into finding undominated solutions to a multi-objective fuzzy linear program. We also do not know of an algorithm that will produce undominated solutions to a multi-objective fuzzy linear program. So, we will employ a directed search method, called an evolutionary algorithm, to find (approximate) undominated solutions. Details about the evolutionary algorithm we used are in Section 4.

We consider two applications of the FFLP in Section 5. The first is the classical max problem of finding the best product mix to obtain the highest revenue. The second is the well-known min problem of finding the least cost diet which satisfies certain minimum requirements. In both cases we demonstrate that our evolutionary algorithm produces good approximate solutions.

The last section contains a brief summary, our conclusions, and directions for future research.

In [4] the authors advocated using fuzzy chaos to search for an optimal solution to the FFLP. However, this procedure is very inefficient being similar to pure random search. We decided instead to employ a directed search technique.

To the authors knowledge no research has treated the FFLP. That is, no one has discussed what it means for $\overline{X}$ to be a "solution" to a fuzzy linear program where all the parameters and variables are fuzzy. Many papers have been written where the $\overline{C}_i$ are fuzzy numbers, or the $\overline{A}_{ij}$ and $\overline{B}_j$ are fuzzy numbers. See [13,15] for a survey of fuzzy linear programming. We believe our method of changing the FFLP into a multi-objective fuzzy linear program is an important step in analyzing the FFLP.

2. Maximizing $\overline{Z}$

$\overline{Z}$ will be a triangular-shaped fuzzy number like the one shown in Fig. 1. Let A_1 be the area under the graph from z_1 to z_2 and A_2 the area under the graph from z_2 to z_3. In a max problem we will max z_2 and A_2 and min A_1. The reason for this decision is: (1) we wish to make $\overline{Z}$ as large as possible so we make z_2 (the most probable value) as large as possible; (2) we want the possibility of exceeding z_2 to be large so we max A_2; and (3) we wish the possibility of obtaining less than z_2 to be small so we min A_1. One cannot directly maximize $\overline{Z}$ since it is a fuzzy number. The analogy is from risk theory where $\overline{Z}$ would be the probability density function of a random variable Z we wish to maximize. There one may [7,16] maximize the expected value of Z, minimize the variance of Z and maximize the skewness to the right of the expected value.

So, for $\max \overline{Z}$ we have a multi-objective optimization problem

$$[\sup z_2, \sup A_2, \inf A_1], \tag{6}$$

for feasible $\overline{X}$, where sup = supremum and inf = infimum. We must use sup and inf because there is no guarantee that $\max z_2$, $\max A_2$ or $\min A_1$ exist.