Evolutionary algorithm solution to fuzzy problems:
Fuzzy linear programming

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Abstract

In this paper we wish to find solutions to the fully fuzzified linear program where all the parameters and variables are fuzzy numbers. We first change the problem of maximizing a fuzzy number, the value of the objective function, into a multi-objective fuzzy linear programming problem. We prove that fuzzy flexible programming can be used to explore the whole undominated set to the multi-objective fuzzy linear program. An evolutionary algorithm is designed to solve the fuzzy flexible program and we apply this program to two applications to generate good solutions. © 2000 Elsevier Science B.V. All rights reserved.

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1. Introduction

In this paper we wish to construct solutions to the fully fuzzified linear program (written FFLP)

\[
\begin{align*}
\text{max} \quad & Z = C^\top X \\
\text{s.t.} \quad & A^\top X \preceq B, \quad X \succeq 0,
\end{align*}
\]

where the $C$, $A_{ij}$, and $B_i$ are all triangular fuzzy numbers and the $X_i$ are also triangular fuzzy numbers. We will code the FFLP as

\[
\begin{align*}
\text{max} \quad & Z = \overline{C}X \\
\text{s.t.} \quad & \overline{A}X \leq \overline{B}, \quad X \geq 0,
\end{align*}
\]

where $\overline{C} = (\overline{C}_1, \ldots, \overline{C}_n)$, $\overline{X} = (X_1, \ldots, X_n)$, $\overline{B} = (B_1, \ldots, B_m)$ and $\overline{A} = [A_{ij}]$ a $m \times n$ matrix of fuzzy numbers. Before we discuss the contents of the paper, let us introduce the basic notation to be employed. We place a bar over a capital letter to denote a fuzzy subset of the real numbers. So $\overline{A}$, $\overline{B}$, $\overline{C}$, $\overline{X}$, etc., are all fuzzy subsets of the real numbers. We write $\overline{A}(x)$, a number in $[0, 1]$, for the membership function of $\overline{A}$ evaluated at $x$. An $x$-cut of $\overline{A}$, written as $\overline{A}(x)$, is defined as $\{x \mid \overline{A}(x) \geq x\}$, for $0 < x \leq 1$. We separately

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specify $\mathcal{A}[0]$ as the closure of the union of all the $\mathcal{A}[x]$, $0 < x \leq 1$.

A triangular fuzzy number $\mathcal{N}$ is defined by three numbers $a_1 < a_2 < a_3$ where the graph of $\mathcal{N}(x)$ is a triangle with base on the interval $[a_1, a_3]$ and vertex at $x = a_2$. We specify $\mathcal{N}$ as $(a_1/a_2/a_3)$. The degenerate cases of $a_1 = a_2 < a_3$ and $a_1 < a_2 = a_3$, where we get half of a triangular fuzzy number, are allowed and we still call them triangular fuzzy numbers. A triangular-shaped fuzzy number $\mathcal{M}$ is partially defined by three number $a_1 < a_2 < a_3$ where: (1) the graph of $\mathcal{M}(x)$ is continuous and monotonically increasing from zero to one on $[a_1, a_2]$; (2) $\mathcal{M}(a_2) = 1$; and (3) the graph of $\mathcal{M}(x)$ is continuous and monotonically decreasing from one to zero on $[a_2, a_3]$. We write $\mathcal{M} \geq 0$ if $a_1 \geq 0$, $\mathcal{M} > 0$ if $a_1 > 0$, $\mathcal{M} \leq 0$, if $a_3 \leq 0$ and $\mathcal{M} < 0$ for $a_3 < 0$. We will use standard fuzzy arithmetic, from the extension principle, to evaluate sums, products, etc. of fuzzy numbers. The $x$-cut of a fuzzy number is always a closed and bounded interval.

To properly define the FFLP we must do two things: (1) define what we mean by max $\mathcal{Z}$, or finding the “maximum” of a fuzzy number; and (2) explain what is meant by $\mathcal{E}_i \leq \mathcal{B}_i$, where $\mathcal{E}_i = \mathcal{A}_{i_1}X_1 + \cdots + \mathcal{A}_{i_n}X_n$. How we are going to handle the problem of “maximizing” a fuzzy number is the topic of the next section and in Section 3 we discuss methods of evaluating fuzzy inequalities.

Searching for the optimal $\mathcal{X}$ to the FFLP is a very difficult problem. Let us assume that the meaning of “optimal” has been defined and the FFLP has an optimal solution. Then we do not know of an algorithm that will compute the exact value of the optimal $\mathcal{X}$. In the next section we turn the search for the optimal $\mathcal{X}$ into finding dominated solutions to a multi-objective fuzzy linear program. We also do not know of an algorithm that will produce dominated solutions to a multi-objective fuzzy linear program. So, we will employ a directed search method, called an evolutionary algorithm, to find (approximate) dominated solutions. Details about the evolutionary algorithm we used are in Section 4.

We consider two applications of the FFLP in Section 5. The first is the classical max problem of finding the best product mix to obtain the highest revenue. The second is the well-known min problem of finding the least cost diet which satisfies certain minimum requirements. In both cases we demonstrate that our evolutionary algorithm produces good approximate solutions.

The last section contains a brief summary, our conclusions, and directions for future research.

In [4] the authors advocated using fuzzy chaos to search for an optimal solution to the FFLP. However, this procedure is very inefficient being similar to pure random search. We decided instead to employ a directed search technique.

To the authors knowledge no research has treated the FFLP. That is, no one has discussed what it means for $\mathcal{X}$ to be a “solution” to a fuzzy linear program where all the parameters and variables are fuzzy. Many papers have been written where the $\mathcal{C}_i$ are fuzzy numbers, or the $\mathcal{A}_{ij}$ and $\mathcal{B}_j$ are fuzzy numbers. See [13,15] for a survey of fuzzy linear programming. We believe our method of changing the FFLP into a multi-objective fuzzy linear program is an important step in analyzing the FFLP.

2. Maximizing $\mathcal{Z}$

$\mathcal{Z}$ will be a triangular-shaped fuzzy number like the one shown in Fig. 1. Let $A_1$ be the area under the graph from $z_1$ to $z_2$ and $A_2$ the area under the graph from $z_2$ to $z_1$. In a max problem we will max $z_2$ and $A_2$ and min $A_1$. The reason for this decision is: (1) we wish to make $\mathcal{Z}$ as large as possible so we make $z_2$ (the most probable value) as large as possible; (2) we want the possibility of exceeding $z_2$ to be large so we max $A_2$; and (3) we wish the possibility of obtaining less than $z_2$ to be small so we min $A_1$. One cannot directly maximize $\mathcal{Z}$ since it is a fuzzy number. The analogy is from risk theory where $\mathcal{Z}$ would be the probability density function of a random variable $\mathcal{Z}$ we wish to maximize. There one may [7,16] maximize the expected value of $\mathcal{Z}$, minimize the variance of $\mathcal{Z}$ and maximize the skewness to the right of the expected value.

So, for max $\mathcal{Z}$ we have a multi-objective optimization problem

$$[\sup z_2, \sup A_2, \inf A_1],$$

for feasible $\mathcal{X}$, where $\sup = \text{supremum}$ and $\inf = \text{infimum}$. We must use sup and inf because there is no guarantee that max $z_2$, max $A_2$ or min $A_1$ exist.
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