Fuzzy Sets and Systems 109 (2000) 21–33
FUZZY sets and systems
www.elsevier.com/locate/fss

Linear programming with fuzzy variables

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Received January 1997; received in revised form February 1998

Abstract

In practice, there are many problems in which all decision parameters are fuzzy numbers, and such problems are usually solved by either possibilistic programming or multiobjective programming methods. Unfortunately, all these methods have shortcomings. In this note, using the concept of comparison of fuzzy numbers, we introduce a very effective method for solving these problems. Then we propose a new method for solving linear programming problems with fuzzy variables.

Keywords: Linear programming; Multiobjective programming; Fuzzy number

1. Introduction

Bellman and Zadeh [2] proposed the concept of decision making in fuzzy environments. Lai and Hwang [7], Tong Shaocheng [15], Buckley [3,4], Negi [8] among others, considered the situation where all parameters are fuzzy. Lai and Hwang assume that the parameters have a triangular possibility distribution. They use an auxiliary model which is solved by multiobjective linear programming methods. Tong Shaocheng defines a goal for the objective function and then by using Zadeh's "Min" operator obtains an optimal solution. Buckley and Negi obtain an optimal solution by using possibility concepts.

In this note, using the concept of comparison of fuzzy numbers, we propose a new method for solving fuzzy number linear programming problems. There are different methods for comparison of fuzzy numbers [1,5,6,10,12,13]. One of the most convenient methods is comparison by integration [1,6,10]. In Section 2, we introduce fuzzy numbers and some of the results of applying fuzzy arithmetic on them and also comparison of fuzzy numbers by Roubens's method [6,14]. In Section 3, we propose a method for solving fuzzy number programming problems. In Section 4, we introduce a linear programming problem with fuzzy variables and propose a method for solving this problem. This paper extends the method proposed in [9].

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0165-0114/00/$ – see front matter
PII: S0165-0114(98)00066-9

2. Preliminaries

Let $\tilde{A}$ be a fuzzy number, i.e. a convex normalized fuzzy subset of the real line in the sense that:

(a) $\exists x_0 \in R$ and $\mu_{\tilde{A}}(x_0)=1$, where $\mu_{\tilde{A}}(x)$ is the membership function specifying to what degree x belongs to $\tilde{A}$.

(b) $\mu_{\tilde{A}}$ is a piecewise continuous function.

The α-level set of $\tilde{A}$ is the set

$$\tilde{A}_\alpha = \{x \in R \mid \mu_{\tilde{A}}(x) \geqslant \alpha\},$$

where $\alpha \in (0,1]$. The lower and upper bounds of any α-level set $\tilde{A}_\alpha$ are represented by $\inf_{x\in\tilde{A}_\alpha}$ and $\sup_{x\in\tilde{A}_\alpha}$ and we suppose that both are finite.

A function, usually denoted by "L" or "R", is a reference function of a fuzzy number iff

(1) $L(x)=L(-x)$,

(2) $L(0)=1$,

(3) L is nonincreasing on $[0,+\infty)$.

A convenient representation of fuzzy numbers is in the form of an L–R flat fuzzy number which is defined as

$$\mu_{\tilde{A}}(x)=\begin{cases} L((A^{\mathrm{L}}-x)/\alpha) & \text{if } x \leqslant A^{\mathrm{L}},\ \alpha>0,\\ R((x-A^{\mathrm{U}})/\beta) & \text{if } x \geqslant A^{\mathrm{U}},\ \beta>0,\\ 1 & \text{otherwise,}\end{cases}$$

where $A^{\mathrm{L}}<A^{\mathrm{U}}$, $[A^{\mathrm{L}},A^{\mathrm{U}}]$ is the core of $\tilde{A}$, $\mu_{\tilde{A}}(x)=1\ \forall x\in[A^{\mathrm{L}},A^{\mathrm{U}}]$, $A^{\mathrm{L}},A^{\mathrm{U}}$ are the lower and upper modal values of $\tilde{A}$ and $\alpha>0$, $\beta>0$ are the left-hand and right-hand spreads [14].

More briefly, a flat fuzzy number is denoted by $\tilde{A}=[A^{\mathrm{L}},A^{\mathrm{U}},\alpha,\beta]_{LR}$.

Among the various type of L–R flat fuzzy numbers, trapezoidal fuzzy numbers, denoted by $\tilde{A}=(A^{\mathrm{L}},A^{\mathrm{U}},\alpha,\beta)$, are of the greatest importance [14].

Let $\tilde{a}=(a^{\mathrm{L}},a^{\mathrm{U}},\alpha,\beta)$ and $\tilde{b}=(b^{\mathrm{L}},b^{\mathrm{U}},\gamma,\theta)$ both be trapezoidal fuzzy numbers. Some of the results of applying fuzzy arithmetic on the fuzzy numbers $\tilde{a}$ and $\tilde{b}$ follow:

Scalar multiplication:

$$x>0,\quad x\in R:\ x(.)\,\tilde{a}=(xa^{\mathrm{L}},xa^{\mathrm{U}},x\alpha,x\beta),$$

$$x<0,\quad x\in R:\ x(.)\,\tilde{a}=(xa^{\mathrm{U}},xa^{\mathrm{L}},-x\beta,-x\alpha).$$

Addition:

$$\tilde{a}+\tilde{b}=(a^{\mathrm{L}}+b^{\mathrm{L}},a^{\mathrm{U}}+b^{\mathrm{U}},\alpha+\gamma,\beta+\theta).$$

Subtraction:

$$\tilde{a}-\tilde{b}=(a^{\mathrm{L}}-b^{\mathrm{U}},a^{\mathrm{U}}-b^{\mathrm{L}},\alpha+\theta,\beta+\gamma).$$

The main concept of comparison of fuzzy numbers is based on the compensation of areas determined by the membership functions [1,10].

Let $\tilde{a},\tilde{b}$ be fuzzy numbers and $S_L(\tilde{a},\tilde{b})$, $S_R(\tilde{a},\tilde{b})$ be the areas determined by their membership functions according to the formulae:

$$S_L(\tilde{a},\tilde{b})=\int_{I(\tilde{a},\tilde{b})}(\inf\tilde{a}_\alpha-\inf\tilde{b}_\alpha)\,\mathrm{d}\alpha,$$