



ELSEVIER

Fuzzy Sets and Systems 109 (2000) 3–19

FUZZY
sets and systems

www.elsevier.com/locate/fss

Interactive fuzzy programming for multi-level linear programming problems with fuzzy parameters

Masatoshi Sakawa^{a,*}, Ichiro Nishizaki^a, Yoshio Uemura^b

^a*Department of Industrial and Systems Engineering, Faculty of Engineering, Hiroshima University, 1-4-1 Kagamiyama, Higashi-Hiroshima 739-8527, Japan*

^b*Products Projects Development Department, Juken Sangyou Co., Ltd., 1-1 Mokuzaikouminami, Hatsukaichi, Hiroshima 738-0022, Japan*

Received July 1997; received in revised form April 1998

Abstract

This paper presents interactive fuzzy programming for multi-level linear programming problems with fuzzy parameters. In fuzzy programming for multi-level linear programming problems, recently developed by Lai et al., since the fuzzy goals are determined for both an objective function and decision variables at the upper level, undesirable solutions are produced when these fuzzy goals are inconsistent. In order to overcome such problems, after eliminating the fuzzy goals for decision variables, interactive fuzzy programming for multi-level linear programming problems with fuzzy parameters is presented. In our interactive method, after determining the fuzzy goals of the decision makers at all levels, a satisfactory solution is derived efficiently by updating the satisfactory degrees of decision makers with considerations of overall satisfactory balance among all levels. Illustrative numerical examples for two-level and three-level linear programming problems are provided to demonstrate the feasibility of the proposed method. © 2000 Elsevier Science B.V. All rights reserved.

Keywords: Multilevel linear programming problem with fuzzy parameters; Fuzzy programming; Fuzzy goals; Interactive methods

1. Introduction

Two-level programming problems, in which a decision maker (DM) at the upper level makes a decision subject to an optimization problem for a DM at the lower level, have two interpretations. They depend on whether there is a cooperative relationship among DMs or not.

Consider a decision problem in a decentralized firm as an example of a decision problem with cooperative

DMs. Top management, an executive board, or headquarters interests itself in overall management policy such as long-term corporate growth or market share. In contrast, operation divisions of the firm are concerned with coordination of daily activities. After headquarters make a decision in accordance with the overall management policy, each division determines a goal to be achieved and tries to attain the goal, fully understanding the decision by the headquarters.

As an example of a decision problem without cooperative DMs, consider the Stackelberg duopoly: Firms 1 and 2 supply homogeneous goods to a market. Suppose Firm 1 dominates Firm 2 in the market, and

* Corresponding author.

E-mail address: sakawa@msl.sys.hiroshima-u.ac.jp (M. Sakawa)

consequently Firm 1 first determines a level of supply and then Firm 2 decides its level of supply after it realizes Firm 1's level of supply.

There is essentially a cooperative relationship between the DM at the upper level and the DM at the lower level in the former problem while each DM does not have a motivation to cooperate each other in the latter problem.

As the former's mathematical programming problem, we can model such a problem as a single-objective large-scale mathematical programming problem used the decomposition method or a multi-objective programming problem with objective functions of all levels. The two-level programming formulation is intended to supplement the decomposition approach, not supplant it [4]. However, the formulation is noteworthy because a hierarchical structure of the decision problem is explicitly included in a mathematical model.

Studies on the latter have been seen in the literature on game theory. Such a situation is modeled as a Stackelberg game, in which there are two players, and one player determines a strategy and thereafter the other player decides a strategy [9]. Each player completely knows objective functions and constraints of an opponent and himself/herself, and the DM at the upper level (leader) first specifies a strategy and then the DM at the lower level (follower) specifies a strategy so as to optimize the objective with full knowledge of the decision of the DM at the upper level. According to the rule, the DM at the upper level also specifies the strategy so as to optimize the objective. Then a solution defined as the above-mentioned procedure is called the Stackelberg strategy (solution).

The Stackelberg strategy has been employed as a solution concept when decision problems are modeled as two-level programming problems, whether there is a cooperative relationship between the DMs or not. Even if the objective functions of both DMs and the common constraint functions are linear, it is known that this problem is a non-convex programming problem with a special structure. In general, the Stackelberg solution does not satisfy Pareto optimality because of its non-cooperative nature.

Computation methods for the Stackelberg solution are classified roughly into three categories: the vertex enumeration approach based on a characteristic that an extreme point of a set of best responses of the DM

at the lower level is also that of a set of the common constraints, the Kuhn–Tucker approach in which the upper level's problem with constraints including optimality conditions of the lower level's problem is solved, and the penalty function approach which adds a penalty term to the upper level's objective function so as to satisfy optimality of the lower level's problem.

The K th best method proposed by Bialas and Karwan [4] is one of vertex enumeration approaches. The solution search procedure of the method starts from a point which is an optimal solution to the problem of the upper level and checks whether it is also an optimal solution to the problem of the lower level or not. If the first point is not the Stackelberg solution, the procedure continues to examine the second best solution to the problem of the upper level and so on. The Kuhn–Tucker method is used by Bialas and Karwan [4] in their parametric complementary pivot algorithm. Bard and Falk [3] replaces the complementarity constraint (complementary slackness condition) with a separable representation and applies a general branch and bound algorithm. Bard [1] formulates a two-level programming problem as an equivalent semi-infinite problem and develops his grid search algorithm through a parametric linear program technique. Ünlü [10] proposes an algorithm based on bicriteria programming by using the result of Bard [1]. White and Anandalingam [13] develops an approach to two-level programming using a duality gap-penalty function format.

For obtaining the Stackelberg solution to a multi-level linear programming problem, Bard [2] and Wen and Bialas [11] propose algorithms for three-level problems. Bard [2] formulates a normal nonlinear programming problem by using the Kuhn–Tucker conditions for the problems of the third level and the second level, and proposes a cutting plane algorithm employing a vertex search procedure to solve a three-level linear programming problem. Wen and Bialas [11] develop a hybrid algorithm to solve a three-level linear programming problem. The algorithm adopts the K th best algorithm to generate the K th best extreme point and the complementary pivot algorithm to check feasibility.

Recently, Lai [5] and Shih, Lai and Lee [8] have proposed a solution concept, which is different from the concept of a Stackelberg solution, for problems such that decisions of DMs in all the levels are sequen-

متن کامل مقاله

دریافت فوری ←

ISIArticles

مرجع مقالات تخصصی ایران

- ✓ امکان دانلود نسخه تمام متن مقالات انگلیسی
- ✓ امکان دانلود نسخه ترجمه شده مقالات
- ✓ پذیرش سفارش ترجمه تخصصی
- ✓ امکان جستجو در آرشیو جامعی از صدها موضوع و هزاران مقاله
- ✓ امکان دانلود رایگان ۲ صفحه اول هر مقاله
- ✓ امکان پرداخت اینترنتی با کلیه کارت های عضو شتاب
- ✓ دانلود فوری مقاله پس از پرداخت آنلاین
- ✓ پشتیبانی کامل خرید با بهره مندی از سیستم هوشمند رهگیری سفارشات