ELSEVIER

Fuzzy Sets and Systems 111 (2000) 55–69

FUZZY sets and systems

www.elsevier.com/locate/fss

An interactive fuzzy satisficing method for multiobjective block angular linear programming problems with fuzzy parameters

Masatoshi Sakawa*, Kosuke Kato

Department of Industrial and Systems Engineering, Faculty of Engineering, Hiroshima University, Higashi-Hiroshima 739-8527, Japan

Received October 1998

Abstract

In this paper, by considering the experts' imprecise or fuzzy understanding of the nature of the parameters in the problem-formulation process, large-scale multiobjective block-angular linear programming problems involving fuzzy parameters characterized by fuzzy numbers are formulated. Using the α-level sets of fuzzy numbers, the corresponding nonfuzzy α-programming problem is introduced. The fuzzy goals of the decision maker for the objective functions are quantified by eliciting the corresponding membership functions including nonlinear ones. Through the introduction of an extended Pareto optimality concept, if the decision maker specifies the degree α and the reference membership values, the corresponding extended Pareto optimal solution can be obtained by solving the minimax problems for which the Dantzig–Wolfe decomposition method is applicable. Then a linear programming-based interactive fuzzy satisficing method for deriving a satisficing solution for the decision maker efficiently from an extended Pareto optimal solution set is presented along with an illustrative numerical example. © 2000 Elsevier Science B.V. All rights reserved.

Keywords: Decomposition methods; Block angular structure; Large-scale linear programs; Fuzzy programming; Interactive methods

---

1. Introduction

In multiobjective optimization problems, multiple objectives are often noncommensurable and cannot be combined into a single objective. Moreover, the objectives usually conflict with each other in that any improvement of one objective can be achieved only at the expense of another. Consequently, in multiobjective optimization, the notion of Pareto optimality or efficiency has been introduced instead of the optimality concept for single-objective optimization. However, decisions with Pareto optimality or efficiency are not uniquely determined; the final decision must be selected from among the set of Pareto optimal or efficient solutions [2, 8, 15].

On the other hand, almost all of the optimization problems facing humans today have very large numbers of variables. Fortunately, however, most of the large-scale programming problems arising in real-world

---

* Corresponding author. Tel.: +81-82-424-7694; fax:+81-82-424-7694.
E-mail address: sakawa@msl.sys.hiroshima-u.ac.jp (M. Sakawa)

PII: S0165-0114(98)00452-7

application almost always have a special structure that can be exploited. One familiar structure is the block angular structure to the constraints and decomposition methods have several significant advantages over conventional methods in solving large-scale programming problems with the block angular structure. From such a point of view, since Dantzig and Wolfe [3] proposed the decomposition principle for large-scale linear programming problems with block angular structure at the beginning of 1960's, subsequent works on large-scale mathematical programming problems with block angular structure have been numerous [3, 8–14, 24, 21, 22].

At this juncture, as a first attempt, we focused on large-scale linear programming problems with the block angular structure for which the decision maker is assumed to have a fuzzy goal and fuzzy constraints [1]. Having elicited the linear membership functions which well represent the fuzzy goal and fuzzy constraints, if the convex fuzzy decision [1] was adopted for combining them, it was shown that, under some appropriate conditions, the formulated problem can be reduced to a number of independent linear subproblems and the satisficing solution for the decision maker is directly obtained just only solving the subproblems [20]. Then we also considered a fuzzy programming approach [25, 26, 15], in the framework of the fuzzy decision of Bellman and Zadeh [1], to large-scale multiobjective linear programming problems with block angular structure by incorporating the Dantzig–Wolfe decomposition method. By extending the framework of the fuzzy decision of Bellman and Zadeh, we have proposed an interactive fuzzy satisficing method [19] through the combination of the desirable features of both the interactive fuzzy satisficing methods [15] and the Dantzig–Wolfe decomposition method [3]. Furthermore, in contrast to the large-scale multiobjective block-angular linear programming problems, by considering the experts' imprecise or fuzzy understanding of the nature of the parameters in the problem-formulation process, we have formulated large-scale multiobjective block-angular linear programming problems involving fuzzy parameters characterized by fuzzy numbers. Using the α-level sets of fuzzy numbers, we [18] have introduced the α-Pareto optimality concepts and presented an interactive decision-making method which utilizes the Dantzig–Wolfe decomposition method.

Under these circumstances, in this paper, we further consider large-scale multiobjective block-angular linear programming problems involving fuzzy parameters by incorporating the fuzzy goals of the decision maker. These fuzzy parameters, reflecting the experts' ambiguous understanding of the nature of the parameters in the problem-formulation process, are assumed to be characterized as fuzzy numbers. Using the α-level sets of fuzzy numbers, the corresponding nonfuzzy α-programming problem is introduced. The fuzzy goals of the decision maker for the objective functions are quantified by eliciting the corresponding membership functions including nonlinear ones. In our interactive decision making method, if the decision maker specifies the degree α and the reference membership values, the corresponding extended Pareto optimal solution can be obtained by solving the minimax problems for which the Dantzig–Wolfe decomposition method is applicable. The satisficing solution for the DM can be derived efficiently from extended Pareto optimal solutions by updating the reference membership values and/or the degree α based on the current values of the extended Pareto optimal solution together with the trade-off information between the membership functions and the degree α. An illustrative numerical example is provided to demonstrate the feasibility of the proposed method.

2. Problem formulation

In general, a large-scale multiobjective linear programming (LSMOLP) problem with the block angular structure is formulated as:

$$\begin{aligned}
&\text{minimize} \quad \boldsymbol{c}_1\boldsymbol{x} = \boldsymbol{c}_{11}\boldsymbol{x}_1 + \cdots + \boldsymbol{c}_{1p}\boldsymbol{x}_p \\
&\text{minimize} \quad \boldsymbol{c}_2\boldsymbol{x} = \boldsymbol{c}_{21}\boldsymbol{x}_1 + \cdots + \boldsymbol{c}_{2p}\boldsymbol{x}_p \\
&\qquad\qquad\qquad \vdots
\end{aligned}$$