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Using ranking functions in multiobjective fuzzy linear programming [1]

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Abstract

Multiobjective mathematical programming problems, in particular the vector optimization problems, define a well known and studied area because of its relevance to numerous practical applications. In this paper vector optimization problems with a fuzzy nature are considered. In these problems usually it is assumed that all the objective functions involved come from the same decision maker. The problem considered here assumes, however, that the objective functions can be defined by different decision makers, and that the coefficients in each of these objective functions are fuzzy numbers. Hence, solution methodologies for these multiobjective fuzzy mathematical programming problems, using different ordering methods ranking fuzzy numbers, are proposed. As an illustration a bi-objective model for land use is presented. © 2000 Elsevier Science B.V. All rights reserved.

Keywords: Fuzzy linear programming; Vector optimization problem; Fuzzy numbers; Ordering methods

1. Introduction

Making decisions involving multiple objectives is a daily task for a lot of people in the more diverse fields, and hence multiple objective decision making problems have defined a very well-studied topic in the general area of decision making theory. In particular, multiobjective decision making problems which can be modeled as mathematical programming (MP) problems also are very well known.

The involvement of different kinds of fuzziness in these problems is a matter which also has received a great deal of work since the early 1980s, as it is very frequent that decision makers have some lack of precision in stating some of the parameters involved in the model [3,4,6,7,11,12,15].

In this paper we consider a linear multiobjective mathematical programming problem in which the coefficients defining the objective functions are given as fuzzy numbers, and where moreover, each objective can be defined by a different decision maker, with which the respective ways of comparing the fuzzy numbers involved are to be taken into account in order to give operational methodologies for solving the problem.

Consequently in Section 2, the classical vector optimization problem is briefly surveyed as the conceptual basis. Then fuzzy multiobjective optimization

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problems are introduced and described. In Section 4, the focus is on the methodologies to solve the case of fuzzy coefficients in the objective functions. Finally in Section 5, an illustrative example for land use, from [8], is presented and then addressed in accordance with the previously shown methodology approach.

2. Multiobjective mathematical programming problems

Typically, a MP problem is concerned with solving a model like,

$$\begin{array}{ll}\text{Min} & f(x)\\ \text{s.t.} & x\in X,\end{array}\qquad (1)$$

where x is an N-vector of decision variables, f is a real-valued function, usually called objective function, and X is a constraint set,

$$X=\{x/x\in\mathbb{R}^N,\ h_i(x)=0,\ i=1,2,\ldots,p;\ g_j(x)\leqslant 0,\ j=1,2,\ldots,q\},$$

where h_i and g_j are real-valued functions defined on X.

In order to solve the problem one needs to find $x^*\in X$ such that,

$$f(x^*)\leqslant f(x),\quad \forall x\in X$$

and then x^* is called a global optimum of (1).

There are, however, many practical situations which involve multiple objectives. An important class of multiobjective decision problems is the well-known vector optimization problem (VOP) or multiobjective optimization problem. Key works on this subject are due to Pareto [10], introducing the concept of noninferior solution, Kuhn and Tucker [9], giving necessary and sufficient conditions for non-inferiority, Zadeh [14], in referring to a solution of a VOP as noninferior, Chankong and Haimes [5], providing an account of multiobjective theories and methodologies, and many others.

A VOP is addressed as

$$\begin{array}{ll}\text{Min} & [f_1(x),\ldots,f_n(x)]\\ \text{s.t.} & x\in X.\end{array}$$

In particular, when the linear case is considered, that is, when $f_1,\ldots,f_n$, h_i, g_j are linear, as it will be in this paper, the model becomes a linear VOP which is typically stated as

$$\begin{array}{ll}\text{Min} & [c_1x, c_2x,\ldots,c_nx]\\ \text{s.t.} & Ax\leqslant b,\ x\geqslant 0\end{array}\qquad (2)$$

where c_j, $j=1,\ldots,n$ is an N vector of cost coefficients, A an $m\times N$-coefficients matrix of constraints (the technological matrix), and b an m-vector of demand (resource) availability. In the following, for the sake of simplicity, we will refer this linear VOP simply as VOP.

Solving a VOP implies finding its set of noninferior solutions, that is, the set of x^* such that there exists no other $x\in X$ such that $f_j(x)\leqslant f_j(x^*)$ for all $j=1,\ldots,n$ with strict inequality for at least one j. From a practical point of view, however, one needs to relate this concept to an operational one, and the best-known way is to characterize the noninferior solutions as optimal solutions. There are two main approaches to do it: the weighting approach and the constraint approach.

By means of the first approach [5], the weighting problem is defined for some vector of weights as,

$$\begin{array}{ll}\text{Min} & \displaystyle\sum_{j=1}^{n} w_j(c_jx)\\ \text{s.t.} & Ax\leqslant b,\ x\geqslant 0,\end{array}\qquad (3)$$

where $w_j\geqslant 0$ and $\sum_{j=1}^n w_j=1$. In this linear case, as it is well known, all noninferior solutions can be found by solving (3).

The auxiliary problem corresponding to the second approach is usually called the kth-objective λ-constraint problem, [5], and it is addressed as follows:

$$\begin{array}{ll}\text{Min} & c_kx\\ \text{s.t.} & c_jx\leqslant\lambda_j,\ j=1,\ldots,n,\ j\neq k,\\ & Ax\leqslant b,\ x\geqslant 0,\end{array}\qquad (4)$$

where $\lambda=(\lambda_1,\ldots,\lambda_{k-1},\lambda_{k+1},\ldots,\lambda_n)$. In this case all noninferior solutions can be found by solving the constraint problem (4).

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