Fuzzy Sets and Systems 111 (2000) 3–28

FUZZY sets and systems

www.elsevier.com/locate/fss

Possibilistic linear programming: a brief review of fuzzy mathematical programming and a comparison with stochastic programming in portfolio selection problem

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Received October 1998

Abstract

In this paper, we review some fuzzy linear programming methods and techniques from a practical point of view. In the first part, the general history and the approach of fuzzy mathematical programming are introduced. Using a numerical example, some models of fuzzy linear programming are described. In the second part of the paper, fuzzy mathematical programming approaches are compared to stochastic programming ones. The advantages and disadvantages of fuzzy mathematical programming approaches are exemplified in the setting of an optimal portfolio selection problem. Finally, some newly developed ideas and techniques in fuzzy mathematical programming are briefly reviewed.

Keywords: Fuzzy mathematical programming; Fuzzy constraint; Fuzzy goal; Possibility measure; Necessity measure; Simplex method; Stochastic programming; Portfolio selection

1. Introduction

The notion of fuzzy set is widely spread to various fields after a resounding success in the applications of fuzzy logic controllers in late 1980s. The application to mathematical programming has relatively long history (see [107]). In spite of the fact that there is no big boom in applications of fuzzy sets theory to the mathematical programming, the history of fuzzy mathematical programming is rich enough. This is the fruit of the continuous efforts of the researchers in that topic. Therefore, it is not easy to describe all of the fuzzy mathematical programming techniques in one paper.

In this paper, we restrict ourselves to describing the essence of fuzzy mathematical programming, especially possibilistic linear programming and to demonstrating its characteristics by using concrete examples, instead of introducing a lot of fuzzy mathematical programming techniques. The readers who are interested in various fuzzy mathematical programming techniques are referred to Slowinski [94], Luhandjula [60], Inuiguchi et al. [33], Rommelfanger [89], Sakawa [92] and Lai-Hwang [56,57].

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[1] This research was partly supported by the Czech Grant Agency, Grant No. 201/98/0222.

0165-0114/00/$ - see front matter
PII: S0165-0114(98)00449-7

In the first part of the paper, we introduce an illustrative realistic example in order to explain why the fuzzy mathematical programming problem is developed. Here it is emphasized that two kinds of uncertainty, ambiguity and vagueness are treated in the fuzzy mathematical programming. A general fuzzy mathematical programming approach is described. After this general description, a fuzzy mathematical programming technique is applied to a concrete realistic example in the succeeding sections. In this part, the required knowledge for developing the method is also explained; moreover, in order to emboss the characteristics of the fuzzy mathematical programming approach, the difference from the conventional mathematical programming approach is examined. In the second part of the paper, as the fuzzy mathematical programming approach is similar to the stochastic programming approach, those approaches are compared using the simple programming problem – portfolio selection problem. The advantages and disadvantages of the fuzzy mathematical programming approach over the stochastic programming approach are highlighted. Finally, some new approaches are briefly overviewed.

Part I: Methods and Techniques

2. Fuzzy mathematical programming

2.1. Fuzzy mathematical programming problem

Let us consider the following production planning problem (from Inuiguchi et al. [44]):

Example 1. There is a factory where two products P and Q are manufactured by two processes M and N. It takes *about* 2 *min* at Process M and *about* 6 *min* at Process N for manufacturing a batch of Product P. On the other hand, it takes *about* 3 *min* at Process M and *about* 4 *min* at Process N for manufacturing a batch of Product Q. It is desired that the working time of Process M (resp. N) is *substantially smaller than* 900 (resp. 1800) min per one term. The profit rates ($/batch) of Products P and Q are *about 7* and *about 9*, respectively. The prices ($/batch) of Products P and Q are *about* 60 and *about* 45, respectively. The factory manager requires the gross sales *substantially larger than* $22 000. Moreover, he wants to have the possibility of profit *substantially larger than* $3400. How many Products P and Q should be manufactured under such circumstances?

This problem is not clearly described as it includes uncertainty in the italic and slanted descriptions. As pointed out by some researchers (see [10,54]), two major different kinds of uncertainties, *ambiguity* and *vagueness* exist in the real life. While *ambiguity* is associated with one-to-many relations, that is, situations in which the choice between two or more alternatives is left unspecified, *vagueness* is associated with the difficulty of making sharp or precise distinctions in the world; that is, some domain of interest is vague if it cannot be delimited by sharp boundaries (see [54]).

In the above example, the slanted uncertain descriptions show the ambiguities of the true values, e.g., *about* 2 *min* shows that one value around 2 is true but not known exactly. On the other hand, the italic uncertain descriptions show the vagueness of the aspiration levels, e.g., *substantially smaller than* 900 *min* does not define a sharp boundary of a set of satisfactory values but shows that values around 900 and smaller than 900 are to some extent and completely satisfactory, respectively.

The fuzzy mathematical programming is developed for treating such uncertainties in the setting of optimization problems. The fuzzy mathematical programming can be classified into three categories in view of the kinds of uncertainties treated in the method (see [44]);

1. fuzzy mathematical programming with vagueness,
2. fuzzy mathematical programming with ambiguity,
3. fuzzy mathematical programming with vagueness and ambiguity.

The fuzzy mathematical programming in the first category was initially developed by Bellman and Zadeh [1], Tanaka et al. [103] and Zimmermann [109,110]. It treats decision making problem under fuzzy goals and constraints. The fuzzy goals and constraints represent the flexibility of the target values of objective functions and the elasticity of constraints. From this point of view, this type of fuzzy mathematical programming is called the *flexible programming*. Numerous papers were devoted to the development of this method. Many of them were overviewed by Zimmermann [111].