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The difference between the managerial and mathematical interpretation of sensitivity analysis results in linear programming

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Abstract

This paper shows that managerial questions are not answered satisfactorily with the mathematical interpretation of sensitivity analysis when the solution of a linear programing model is degenerate. Most of the commercially available software packages provide sensitivity results about the optimality of a basis and not about the optimality of the values of the decision variables. The misunderstanding of the shadow price and the validity range information provided by a simplex based computer program may lead to wrong decision with considerable financial losses and strategic consequences. The paper classifies the most important types of sensitivity information, graphically illustrates degeneracy, and demonstrates its effect on sensitivity analysis. A production planning example is provided to show the possibility of faulty production management decisions when sensitivity results are not understood correctly. Finally the recommendations for the users of linear programing models and for software developers are provided.

Keywords: Linear programming; Sensitivity analysis; Production management

1. Introduction

Linear programing (LP) is one of the most extensively used operations research technique in production and operations management [1]. As a result of the development of computer technology and the rapid evolution of user friendly LP software, every operation manager can run an LP software easily and quickly on a laptop computer. Although solving LP models is now accessible for everybody, the interpretation of the results requires a lot of skill. Most of the management science and OR textbooks pay special attention to sensitivity analysis, and the problems of degeneracy, but sensitivity analysis under degeneracy is rarely discussed. Commercially available software do not give enough information to the user about the existence and the consequences of these, very common, "special cases". In practice, managers very frequently misinterpret the LP results which may lead

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to erroneous decisions and to important financial and/or strategic disadvantages.

Several papers have addressed this issue. Evans and Baker draw the attention to the consequences of the misinterpretation of sensitivity analysis results in management. They illustrate their point with a simple example and list some published cases in which the erroneous interpretation of sensitivity analysis results is obvious [2]. Aucamp and Steinberg [3] also warn that shadow price analysis is incorrect in many textbooks, and that the shadow price is not equal to "the optimal solution of the dual problem" when the obtained optimal solution is degenerate. They present some examples of shadow price calculations by commercial packages. Akgül [4] refines the shadow price definition of Aucamp and Steinberg, and introduces the negative and positive shadow prices for the increase and the decrease of the RHS elements. Greenberg [5] shows that very frequently practical LP models have a netform structure, and netform structures are always degenerate. He illustrates sensitivity analysis of netform type models by one of the Midterm Energy Market Model of the U.S. Department of Energy. Gal [6] summarizes most of the critics concerning sensitivity analysis of LP models and highlights some important research directions. Rubin and Wagner [7] illustrates the traps of the interpretation of LP results by using the industry cost curve model in a tutorial type paper written for managers and instructors. Jansen et al. [8] explain the effect of degeneracy on sensitivity analysis by using a transportation model, and present the shortcomings of the most frequently used LP packages. They also show how complete, correct sensitivity analysis can be done. Wendell [9,10] also pays special attention to the correct and practically useful calculation of sensitivity information. The biggest problem is not that operations researchers are unaware of the difficulties of sensitivity analysis. This issue is discussed thoroughly in the scientific literature (see for example [6,10,11]) and a complete, mathematically correct treatment of sensitivity analysis is presented by Jansen et al. [8], and by Roos et al. [12]. Practice, however, shows that the problem is not widely known among the LP users, and the available commercial software packages are not helpful in recognizing the difficulties.

The main objective of this paper is to explain the difference between the managerial questions and the traditional mathematical interpretation of sensitivity analysis. In the first part of the paper basic definitions are introduced, the most important types of sensitivity information are classified, and degenerate LP solutions are illustrated graphically. In the second part a production planning problem is used to demonstrate the consequences of incorrect interpretations of the provided sensitivity information. Finally, some recommendations are made for both practitioners and software developers.

2. Basic definitions and concepts

Every LP problem can be written in the following standard form:

$$\min_x \{c^T x | Ax = b, x \geqslant 0\}, \tag{1}$$

where A is a given $m \times n$ matrix with full row rank and the column vector b represents the right-hand side (RHS) terms and the row vector c^T represents the objective function coefficients (OFC). Problem (1) is called the *primal problem* and a vector $x \geqslant 0$ satisfying $Ax = b$ is called *a primal feasible solution*. The objective is to determine those values of the vector x which minimize the objective function. To every primal problem (1) the following problem is associated:

$$\max_y \{b^T y | A^T y \leqslant c\}. \tag{2}$$

Problem (2) is called the *dual problem* and a vector y satisfying $A^T y \leqslant c$ is called a *dual feasible solution*. For every primal feasible x and dual feasible y it holds that $c^T x \geqslant b^T y$ and the two respective objective function values are equal if and only if both the solutions are optimal.

Most computer programs to solve LP's are based on a version of the simplex method. Modern, hi-performance packages are furnished with interior point solvers as well, however, the