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Real-Time Motion Analysis with Linear Programming[1]

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A method to compute motion models in real time from point-to-line correspondences using linear programming is presented. Point-to-line correspondences are the most reliable measurements for image motion given the aperture effect, and it is shown how they can approximate other motion measurements as well. An error measure for image alignment using the L_1 metric and based on point-to-line correspondences achieves results which are more robust than those for the commonly used L_2 metric. The L_1 error measure is minimized using linear programming. While estimators based on L_1 are not robust in the breakdown point sense, experiments show that the proposed method is robust enough to allow accurate motion recovery over hundreds of consecutive frames. The L_1 solution is compared to standard M-estimators and Least Median of Squares (LMedS) and it is shown that the L_1 metric provides a reasonable and efficient compromise for various scenarios. The entire computation is performed in real-time on a PC without special hardware.

Key Words: motion analysis; linear programming.

1. INTRODUCTION

Robust, real-time recovery of visual motion is essential for many vision-based applications. Numerous methods have been developed for motion recovery from image sequences; among them are algorithms that compute the motion directly from the grey level values or local measures of them [3, 14, 16, 17, 20]. A second class of algorithms use feature points or optical flow to recover motion [1, 9, 15]. A probabilistic error minimization algorithm [26] can be used to recover motion in the presence of outliers. Another class of algorithms use explicit probability distributions of the motion vectors to calculate motion models [23]. Black and Anandan presented [7] a framework for robust calculation of optical flow based

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on the Lorentzian estimator and a Graduated Non-convexity algorithm seeking an optimal solution. Bab-Hadiashar and Suter [4] presented a robust optical flow calculation based on LMedS (Least Median of Squares).

Most of the methods cited above have problems when computing high-order motion models (e.g., an affine motion model or a homography): either they are sensitive to outliers, or the execution speed is slow. Algorithms based on iterative reweighting M-estimators have tuning and initial guess problems, especially in multiple model cases. The LMedS faces complexity and accuracy problems when it is difficult to obtain a good hypothesis by random selection.

In this paper an algorithm to recover high-order motion models from point-to-line correspondences using linear programming is presented. Point-to-line correspondences are robust in the sense that they are largely insensitive to aperture effects and to T-junctions, unlike the common point-to-point correspondences. Point-to-line correspondences can also approximate other measurements as well, such as point-to-point correspondences, correspondences with uncertainty, and the spatiotemporal constraint.

The L_1 metric ($\sum |a_i - b_i|$) can be used with the point-to-line correspondences and is much more robust then the L_2 metric ($\sqrt{\sum (a_i - b_i)^2}$). For example, the median minimizes the L_1 metric, while the centroid (average) minimizes the L_2 metric. L_1 based estimators are not robust in the sense of breakdown point [11, 24, 25] as they are sensitive to leverage points. However, our experiments show that in motion analysis the L_1 error measure is robust enough to compute accurate motion over hundreds of frames, even with large moving outlier objects in the scene. Moreover, this is done in real time on a regular PC (300 MHz).

The linear programming solver does not need an initial guess nor a noise scale estimate, which are required for iterative reweighted least-square algorithms (such as M-estimators). Comparisons between estimators based on the L_1 metric and the robust LMedS estimators show that global motion analysis using the L_1 estimator is only slightly less robust than analysis with the LMedS estimator, but the L_1 computation is much faster.

The motion analysis consists of computing a model based alignment between successive frames. The alignment process consists of two steps: (i) Computing correspondences and representing them as point-to-line correspondences is described in Section 2. (ii) Converting the alignment problem into a linear program using the point-to-line correspondences and solving it is described in Section 3. Section 4 describes experimental results and comparisons with other methods. Section 5 gives concluding remarks. The Appendix describes a possible explanation for the experimental insensitivity of L_1 motion estimators to leverage points.

2. POINT-TO-LINE CORRESPONDENCES

Point-to-line correspondences are used due to their insensitivity to the aperture effect. This section describes the aperture effect, point selection for the point-to-line correspondence, and the use of point-to-line correspondences to represent normal flow and fuzzy correspondences.

2.1. Aperture Effect

A moving line viewed through a small aperture will have an apparent motion which is normal to the line. This phenomena is called the "aperture effect." An example of the aperture effect is shown in Fig. 1. Given the aperture effect we express a constraint on the