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Limit analysis on aging penstocks based on linear programming

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Abstract

For the limit analysis of aging penstocks used in hydraulic power facilities, it would be difficult to apply analytical methods and finite element approaches because of the large size and complex failure mode of the structure, and also due to the cost. The present paper shows that linear programming (LP) could effectively solve these problems, as the limit analysis of an aging penstock is carried out using a combined approach of LP and an analytical solution for the plastic deformation at the limit state. Based on the numerical results, such factors influencing the limit load are discussed as the wall thickness and the span length between anchor blocks. Then, a quantitative evaluation method based on the limit load is proposed for assessing the structural stability of an aging penstock. © 2000 Elsevier Science Ltd. All rights reserved.

Keywords: Penstock; Limit analysis; Linear programming; Structural stability

1. Introduction

A penstock as shown in Fig. 1 is one of the major constituents of a hydraulic power facility. A pressure head inside is exerted by continuously running water through a sharp drop that is needed for hydraulic power generation. Since penstocks are always exposed to water, corrosion and wear develop with time and lead to a decrease in the wall thickness, which may affect the structural stability of the penstock in the long-run. To ensure the structural safety of aging penstocks, the allowable stress concept has been employed in the current design codes, and repairs or replacement of an aging penstock are required once a hoop stress exceeds the allowable stresses. This code practice is, however, deemed conservative. In view of the recent increasing demand for cost effectiveness and prolonged service life of penstocks, the need arises for a more reasonable maintenance procedure in hydraulic power facilities. As an alternative to the allowable stress approach, the issue treated here is the limit analysis of cylindrical shells under internal water pressure, focusing on reducing the limit load or plastic collapse load when the wall thickness has decreased due to corrosion. Based on this ultimate strength approach, a quantitative evaluation method with respect to the structural stability of penstocks is proposed. Issues, which are related with local buckling while a penstock is empty and the dynamic stability, are not treated here.

Theoretically, the problem can be solved using any of the following analytical or numerical approaches: analytical solutions; finite element approaches (FEM); and mathematical programming. The analytical approach, as exemplified in the studies by Hodge [1,2], is applicable only to relatively simple problems because an appropriate failure mechanism needs to be specified for particular given boundary, loading, and yield conditions prior to solving the governing non-linear algebraic

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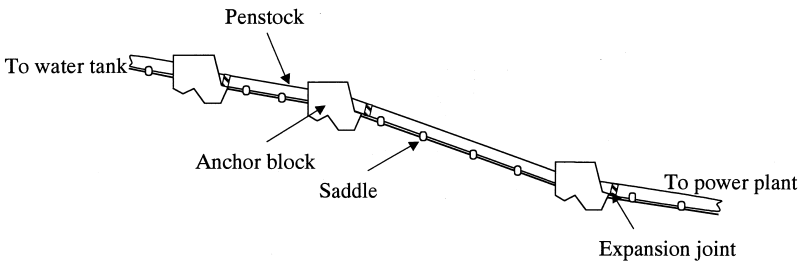

Fig. 1. Penstocks used in hydraulic power facilities.

equations [3–5]. This greatly hinders its applicability to actual structures like penstocks, which are typically supported by anchor blocks several tens of meters apart, and of which the design wall thickness often varies even in the same support section. Although these structural aspects of penstocks could be modeled using a three-dimensional FEM, the real problem in employing FEM lies in the appropriateness in choosing this kind of costly method, considering the sheer scale of the problem. Tin-Loi and Pulmano [6–8] proposed an approach based on mathematical programming and the static theorem of plasticity, which allows this problem to be converted into a linear programming (LP) problem, by discretization and linearization of equilibrium equations, boundary conditions, and yield conditions. Since the method does not require preliminary studies for stress fields or failure mechanisms and the computational time is usually short, a simple and systematic approach can be applied to large-scale structures such as penstocks with complicated modes of failure. However, the available information obtained by this superior approach is so scarce that only the plastic collapse load and internal forces at failure are determined.

In the present study, an effort has been made to extend the scope of solutions by LP to include an analytical solution on the plastic deformation at the limit state, in order to study the failure mechanism of penstocks. Obtaining the plastic collapse load by LP first, then the plastic deformation is calculated by substituting internal forces observed at yielded nodal points into the newly found theoretical formulas. As a practical example, the limit analysis on an aging penstock in a hydraulic power facility is carried out, and the effects of the wall thickness and distance between anchor blocks on the limit load are investigated.

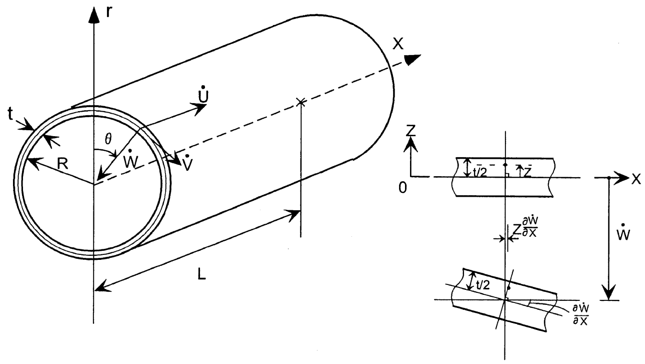

Fig. 2. Shell geometry and the coordinate system.

2. Plastic collapse load

2.1. Equilibrium equations

Fig. 2 shows a penstock between two adjacent anchor blocks with a span length L, a radius R, and a wall thickness t, that is made of a homogeneous and isotropic material with perfectly plastic behavior and subjected only to the internal water pressure. The gravity load of water and the axial force due to temperature changes are omitted. This is because penstocks are usually supported by many saddles besides anchor blocks and thus the bending moment induced by the water weight is minimal. The axial force due to temperature variations is released through expansion joints as shown in Fig. 1. Fig. 3 shows internal forces exerted on a shell element under the internal water pressure P, and a cylindrical coordinate system (X, θ, r). Here, the basic assumptions of the thin shell theory are applied with respect to thickness and displacement. To make the statement clear, the basic equations are briefly introduced below; for details, see Hodge [1].

Taking into consideration the symmetric feature of

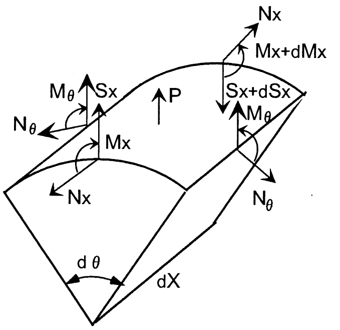

Fig. 3. A shell element.