

# Computational aspects of linear programming Simplex method

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## Abstract

In this paper, the Simplex method is re-examined from the computational view points. Efficient numerical implementation for the Simplex procedure is suggested. Special features of artificial variables, and variables with unrestricted in signs are exploited to reduce the computational efforts, and computer memory requirement. The developed Simplex code has been tested on several examples, and its performance has been compared with existing Simplex codes. © 2000 Elsevier Science Ltd. All rights reserved.

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## 1. Introduction

Any linear programming (LP) problems can [1,2] be expressed in the following standard form.

Find the design variable vector  $\vec{x}$  such that

Minimize

$$F = \vec{c}^T \vec{x} \quad (1)$$

Subject To

$$[A] \vec{x} = \vec{b} \quad (2)$$

and

$$\vec{x} \geq \vec{0} \quad (3)$$

As an example, consider the following simple problem:

Find the vector

$$\vec{x} = \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix}, \text{ such that}$$

Minimize

$$F = -x_1 - 3x_2 \quad (4)$$

Subject to

$$x_1 + x_2 \leq 2 \quad (5)$$

$$2x_1 - x_2 \leq 1 \quad (6)$$

$$x_1, x_2 \geq 0 \quad (7)$$

The inequality constraints shown in Eqs. (5) and (6) can be put in the standard (equality) form by introducing slack and/or surplus variables  $x_3$  and  $x_4$  as following:

$$x_1 + x_2 + x_3 = 2 \quad (8)$$

$$2x_1 - x_2 + x_4 = 1 \quad (9)$$

Eqs. (8) and (9) represent system of two equations and four unknowns. Therefore, there exists infinite number of possible solutions. One possible set of solution is to set

$$x_1 = 0 = x_2 \quad (10)$$

and therefore, one can easily find

$$x_3 = 2 \text{ (= right-hand-side, or rhs, of Eq. (8))}$$

$$x_4 = 1 \text{ (= rhs of Eq. (9)).}$$

In the Simplex method, Eqs. (8) and (9) are said to be in the Canonical form (the columns associated with variables  $x_3$  and  $x_4$  only contain 0's and 1's). Furthermore, the variables with zero values are defined as “Non-Basic” variables, whereas the variables with rhs values are referred to as “Basic” variables. Thus, for the LP problem defined in Eqs. (4)–(7),  $x_3$  and  $x_4$  are the basic variables, and  $x_1$  and  $x_2$  are non-basic variables.

## 2. Simplex method

Observing Eqs. (8) and (9) carefully, one can see that the

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Table 1  
Initial Simplex table

Basic variables	$x_1$	$x_2$	$x_3$	$x_4$	RHS = $b$	$\frac{b}{A}$
$x_3$	1	1	1	0	2	$\frac{2}{1} = 2 \rightarrow$ LEAVE basic group
$x_4$	2	-1	0	1	1	$1/(-1) = -1$
	-1	-3	0	0	Min. $F$	
		↑ ENTER basic group				

number of basic variables should be the same as the number of equality constraints. Furthermore, the total number of designs variables (NDV) can be expressed as:

$$NDV = NBV + NNBV \tag{11}$$

In Eq. (11), NBV and NNBV are defined as the number of basic variables, and the number of non-basic variables, respectively.

The general “key ideas” in the Simplex method can be summarized in the following paragraph:

“In each iteration, one has to properly select a (non-basic) design variable to enter the *basic* (variable) group, and to properly kick out a (basic) variable into a *non-basic* (variable) group”.

2.1. Which non-basic variable should become a basic variable in the next iteration?

To answer the above question, the objective function need be examined carefully. At this current iteration, both variables  $x_1$  and  $x_2$  are in the ‘non-basic group’ (since  $x_1 = 0 = x_2$ ). Since we want to *minimize* the objective function  $F = -x_1 - 3x_2$ , variable  $x_2$  should be selected to become the basic variable (thus, its value will become  $\geq 0$ ) in the next iteration (since it has the largest negative coefficient, it should help the objective function the most)!

2.2. Which basic variable should become a non-basic variable in the next iteration?

We now have two possibilities for basic variables in the next iteration. Either  $x_2$  and  $x_3$ , or  $x_2$  and  $x_4$  should be the basic variables in the next iteration (hence  $x_1$  remains to be non-basic variable, or  $x_1 = 0$ ).

From Eqs. (8) and (9), one has:

$$x_3 = 2 - x_2 \geq 0 \tag{12}$$

$$x_4 = 1 + x_2 \geq 0 \tag{13}$$

The inequality constraints shown in Eqs. (12) and (13) are

Table 2  
The incomplete new (2nd iteration) Simplex table

Basic variables	$x_1$	$x_2$	$x_3$	$x_4$	RHS = $b$	$\frac{b}{A}$
$x_2$		1		0		
$x_4$		0		1		
		0		0		

imposed in order to satisfy the constraint Eq. (3). We do not have to worry about constraint Eq. (13), because it will be automatically satisfied in the *next iteration* (since  $x_2$  had been selected to become basic variable, its value will be either zero, or positive, hence  $1 + x_2$  cannot be a negative number!). Constraint Eq. (12) will be satisfied if  $x_2 \leq 2$ .

Thus, in the next iteration,  $x_2$  will become the basic variable (see Section 2.1) and we would like to see that the value of  $x_2$  to be as large as possible (in order to minimize  $F$ ). However, the largest value  $x_2$  may have is  $x_2 = 2$  (since otherwise constraint Eq. 12 will be violated!).

When  $x_2 = 2$ , the values for  $x_3$  and  $x_4$  can be computed from Eqs. (12) and (13) as:

$$x_3 = 2 - 2 = 0 \tag{14}$$

$$x_4 = 1 + 2 = 3 \tag{15}$$

Since  $x_3 = 0$  and  $x_4 =$  a positive number ( $= 3$ ), it implies that variable  $x_3$  should become a non-basic variable in the next iteration.

In summary, the new basic variables in the next iteration should be  $x_2$  and  $x_4$ . The entire Simplex process (up until now) can be conveniently presented in the following “simplex table” format (see Table 1):

The following comments can be made by observing Table 1.

- (a) Since the columns associated with the basic variables  $x_3$  and  $x_4$  contain only 0’s and 1’s, hence we do have the “canonical” form.
- (b) The first two rows of Table 1 correspond to the original constraint Eqs. (5) and (6), which also include slack/surplus variables  $x_3$  and  $x_4$  (in order to convert 2 inequality constraint Eqs. (5) and (6), into two equality constraint Eqs. (8) and (9)).
- (c) The last row of Table 1 represents the objective function (expressed in terms of the non-basic variables  $x_1$  and  $x_2$ ).
- (d) The last column of Table 1 contains the ratio of the RHS vector  $\vec{b}$  and the column of matrix  $[A]$  which associates with the variable ( $x_2$ ) entering the basic group. The first ratio ( $\vec{b}/A$ ) (see last column of Table 1) represents the fact that  $x_2 \leq 2$  (see Eq. (12)), while the second (*negative*) ratio ( $\vec{b}/A$ ) ( $1/ -1$ , see last column of Table 1) indicates that one needs not worry about the inequality constraint Eq. (13), since it will be automatically satisfied.

In general, one should only consider those ratios ( $\vec{b}/A$ )

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