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Solutions based on fuzzy goals in fuzzy linear programming games

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Abstract

In this paper, we deal with linear programming problems with fuzzy parameters from the viewpoint of experts' imprecise or fuzzy understanding of the nature of parameters in a problem-formulation process and consider fuzzy linear programming games arising from the linear programming problems. Constructing fuzzy goals of coalitions with respect to payoff in order to suitably reflect fuzzy environments in which the linear programming games are formulated, we define solutions maximizing minimal fuzzy goal and solutions maximizing the sum of fuzzy goals and develop computational methods for obtaining the solutions. Finally we give a numerical example to illustrate construction of a fuzzy linear programming game and derivation of the solutions.

Keywords: Linear programming problem with fuzzy parameters; Fuzzy linear programming game; Fuzzy goal; Solution concepts based on fuzzy goals

1. Introduction

By using cooperative game theory, Owen considered linear production programming problems in which multiple decision makers (DMs) pool resources to produce some goods [10]. An objective function of the linear production programming problem was represented as a total revenue from selling some kinds of goods, and the problem was formulated as a linear programming problem in which, subject to resource constraints, the revenue is maximized. He gave an allocation scheme of the total revenue by adopting a point in the core of a cooperative game arising from the production programming problem. Subsequently extensions of the production model and relationship between other optimization problems and the cooperative games have been studied in relation to Owen's work [3–5,7]. When optimization problems with multiple DMs are linear programming problems such as linear production programming problems, cooperative games arising from such linear programming problems are called linear programming (LP) games.

In managerial and public decision making problems, we can find a lot of projects that multiple DMs cooperatively carry out. One of the most important problems in such a project is how to share the total

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costs of the project among the DMs or how to allocate the total revenues. The management is supposed to investigate the problem in the early period of the project and in the latter period. In the early period, the problem is examined to decide whether the project is realized or not, or whether the DM participates in the project or not. In the latter period, it is done to allocate the cost or profit yielding accomplishment of the project. Fuzziness, uncertainty or risk should be introduced in the examination of the problem in the early period of the project because experts have imprecise or fuzzy understanding of the nature of the parameters in the problem-formulation process, and foreseen events might happen before the completion of the project. When the DM decides whether the project is realized or not, or whether the DM participates in the project or not, the result of this paper is utilized in order to estimate their allocations of the profit from the project.

It can be observed here that, in most real-world situations, the possible values of parameters of mathematical models are often only imprecisely or ambiguously known to the experts. With this observation in mind, it would be certainly more appropriate to interpret the experts' understanding of the parameters as fuzzy numerical data which can be represented by means of fuzzy sets of the real line known as fuzzy numbers. The resulting mathematical programming problem involving fuzzy parameters would be viewed as a more realistic version than the conventional one [12,14].

From this viewpoint, assuming that parameters involved in the objective functions and the constraints of the linear programming problem are characterized by fuzzy numbers, Nishizaki and Sakawa extended the linear production programming game in fuzzy environments, and gave an allocation scheme of the total payoff by adopting the solution which is represented as an interval of payoff in the core of the fuzzy linear production programming game [9].

In the early period of the project, the solution represented as an interval is effective in assessing the possibility of realization of the project or in deciding whether the DM should participate in the project or not. With the development of the project, however, it is necessary to evaluate a definite payoff because of business planning or budget-making.

In this paper we formulate a fuzzy cooperative game arising from a linear programming problem with fuzzy parameters, and construct a fuzzy goal of each coalition with respect to payoff in order to suitably describe the fuzzy environments. Two solution concepts based on fuzzy goals are defined and computational methods for obtaining the solutions are developed. Each solution concept provides a payoff for a player which is represented not as an interval but as a point. The first solution is defined by maximizing the minimal fuzzy goal and the second by maximizing the sum of fuzzy goals. We also develop the computational methods in the case where some coalitions are given priority to the others. Finally a numerical example is given to illustrate construction of the fuzzy cooperative game and derivation of the solutions.

2. Fuzzy linear programming games

Let $N=\{1,\ldots,n\}$ denote a set of all the DMs and S denote a subset of N, which is referred to as a coalition, and $v(S)$ be an optimal value to a linear programming problem:

$$\begin{aligned} &\text{maximize} && c_1x_1+\cdots+c_px_p \\ &\text{subject to} && a_{11}x_1+\cdots+a_{1p}x_p\leqslant b_1(S), \\ & && \vdots \\ & && a_{m1}x_1+\cdots+a_{mp}x_p\leqslant b_m(S), \\ & && x_1,\ldots,x_p\geqslant 0. \end{aligned} \tag{1}$$

A game defined by such a pair (N,v) is called an LP game. For example, Owen [10] considered linear production programming problems in which multiple DMs pool resources to produce some goods. An objective