

## Fuzzy linear programming using a penalty method

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### Abstract

In this paper we begin with a standard form of the linear programming problem. We replace each constant in the problem with a fuzzy number. We then reformat the objective and constraints into an unconstrained fuzzy function by penalizing the objective for possible constraint violations. The range of this fuzzy function lies in the space of fuzzy numbers. The objective is then redefined as optimizing the expected midpoint of the image of this fuzzy function. We show that this objective defines a concave function which, therefore, can be maximized globally. We present an algorithm for finding the optimum. © 2001 Elsevier Science B.V. All rights reserved.

*Keywords:* Fuzzy number; Fuzzy function; Possibility distribution; Linear programming

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### 1. Introduction

In constrained optimization problems where uncertainty is characterized using possibility distributions, it may be unavoidable and/or advantageous to consider solutions that have a non-zero possibility of violating one or more of the constraints. This can be done by considering the cost of a constraint violation in the problem formulation. In the following discussion we examine such a formulation of the linear programming problem where the constant terms in the problem may not be known precisely. To incorporate this type of uncertainty into the model each constant in the problem is replaced with a fuzzy number (fuzzy sets on  $R$  with all  $\alpha$ -cuts closed intervals) where we interpret fuzzy numbers as possibility distributions [13]. Next we incorporate the possibility of a constraint violation into the model by replacing each constraint with a term in the objective function that reduces the objective by the cost of the violation. From this an unconstrained fuzzy function optimization problem arises. The image of this fuzzy function is a fuzzy number [8]. Thus our objective is to, in some sense, find the optimal fuzzy number. To do this we adopt a view of possibility distributions as cumulative subjective probability distributions [7] and assume that our utility for a given

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interval of possible values is its midpoint. Thus we calculate the expected midpoint of a fuzzy number and use this as our basis of comparison. Of course the midpoint is simply one of several ways to obtain an optimal fuzzy number. From this point of view, which is the approach of this paper, the objective is to maximize the expected midpoint of the fuzzy number that represents the possible outcomes for a given action. A gradient ascent algorithm for finding the solution to this problem is developed. We note that fuzzy LP problems with similar crisp relations as studied in this paper have been considered in [5,6].

**2. Notation**

In this paper we will interpret membership degrees as levels of possibility. Throughout this paper a fuzzy subset is denoted by the symbol  $\sim$  over a letter. For example if  $X$  is a set, then  $\tilde{x}$  may be used to denote a fuzzy subset of  $X$  and  $\tilde{x}_\alpha$  will denote the  $\alpha$ -level of possibility ( $\alpha$ -cut) for  $\tilde{x}$ , i.e. it is the crisp set  $\{x | \mu_{\tilde{x}}(x) \geq \alpha\}$  for  $\alpha \in (0, 1]$  and  $\text{cls}\{x | \mu(x) > 0\}$  for  $\alpha = 0$  where  $\text{cls}(A)$  denotes the closure of set  $A$  and  $\mu_{\tilde{x}} : X \rightarrow [0, 1]$  is the membership function of  $\tilde{x}$ . If  $\tilde{x}$  is a fuzzy number, we will let  $\tilde{x}_\alpha = [x_\alpha^-, x_\alpha^+]$  be the closed interval which is the  $\alpha$ -cut for  $\tilde{x}$  where  $x_\alpha^-$  and  $x_\alpha^+$  are its left and right endpoints respectively. We note that the real line is a subset of the space of fuzzy numbers, i.e. a real number is a fuzzy number where every  $\alpha$ -cut is equal to the number. For our purposes here, if  $x$  is a vector of real numbers, then  $\tilde{x}$  will be a vector of fuzzy numbers.

Let  $\tilde{A}$  denote a matrix of fuzzy numbers, i.e. each coefficient,  $\tilde{a}_{ij}$ , is a fuzzy number with  $\alpha$ -cut  $(\tilde{a}_{ij})_\alpha = [(\tilde{a}_{ij})_\alpha^-, (\tilde{a}_{ij})_\alpha^+]$ . Let  $\tilde{A}_\alpha$  be the matrix whose coefficients are the  $\alpha$ -cuts of the coefficients of  $\tilde{A}$ , i.e. it is a matrix of closed intervals. Let  $\tilde{A}_\alpha^-$  and  $\tilde{A}_\alpha^+$  denote the matrices whose coefficients are the left and right endpoints of the entries of  $\tilde{A}_\alpha$ , e.g. the  $ij$ th entry of  $\tilde{A}_\alpha^+$  is  $(\tilde{a}_{ij})_\alpha^+$ . We will let  $(\tilde{A}_i)_\alpha^+$  and  $(\tilde{A}^j)_\alpha^+$  denote the  $i$ th row and the  $j$ th column of matrix  $\tilde{A}_\alpha^+$ , respectively.

Finally we define  $EA(\tilde{x}) = \frac{1}{2} \int_0^1 (x_\alpha^- + x_\alpha^+) d\alpha$ , to be the expected midpoint of fuzzy number  $\tilde{x}$ . The cumulative subjective probability interpretation of possibilities is used for this paper [7].

**3. Problem formulation**

The following is the form of the (crisp) linear programming problem considered herein:

$$\begin{aligned} &\text{Maximize } c^T x \\ &\text{subject to } Ax \leq b, \\ &\qquad\qquad x \geq 0, \end{aligned}$$

where  $c$  and  $x \in R^n$  and  $b \in R^m$ .

If there are uncertainties about any of the components of  $A$  and/or  $b$  the possibility of a constraint violation cannot be avoided unless the problem restricts  $x$  to the worst/best possible case (optimistic, pessimistic lp – see [15]). To take into account the possibility of a constraint violation each constraint is replaced with a penalty term in the objective function together with the corresponding uncertainty in the coefficients. The actual penalty term will be problem dependent though its generic representation is developed and analyzed. For this paper we will treat constraints as resources and assume that if a resource is exceeded it can be replenished at a cost that is linear with respect to the amount of the violation. The incorporation of truly crisp constraints is easily handled within our approach but will not be considered further in this paper. The exception to this is that  $x \geq 0$  is considered a crisp constraint. In other words, we will replace the

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