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Interpretation of inequality constraints involving interval coefficients and a solution to interval linear programming

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Abstract

The modern trend in Operations Research methodology deserves modelling of all relevant vague or uncertain information involved in a real decision problem. Generally, vagueness is modelled by a fuzzy approach and uncertainty by a stochastic approach. In some cases, a decision maker may prefer using interval numbers as coefficients of an inexact relationship. As a coefficient an interval assumes an extent of tolerance or a region that the parameter can possibly take. However, its use in the optimization problems is not much attended as it merits.

This paper defines an interval linear programming problem as an extension of the classical linear programming problem to an inexact environment. On the basis of a comparative study on ordering interval numbers, inequality constraints involving interval coefficients are reduced in their satisfactory crisp equivalent forms and a satisfactory solution of the problem is defined. A numerical example is also given.

Keywords: Interval number; Inequality relation; Mathematical programming

1. Introduction

In conventional mathematical programming, coefficients of problems are usually determined by the experts as crisp values. But in reality, in an imprecise and uncertain environment, it is an unrealistic assumption that the knowledge and representation of an expert are so precise. Hence, in order to develop good Operations Research methodology fuzzy and stochastic approaches are frequently used to describe and treat imprecise and uncertain elements present in a real decision problem. In fuzzy programming problems [2,6,10] the constraints and goals are viewed as fuzzy sets and it is assumed that their membership functions are known. On the other hand, in stochastic programming problems [1,5,9,12] the coefficients are viewed as random variables and it is also assumed that their probability distributions are known. These membership functions and probability distributions play important roles in their corresponding methods. However, in reality, to a decision maker (DM) it is not always easy to specify the membership function or the probability distribution in an inexact environment.

At least in some of the cases, use of an interval coefficient may serve the purpose better. Though by using α-cuts, fuzzy numbers can be degenerated into interval numbers [13], deliberately we keep this concept out of the scope of this paper. Here, an interval number is

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considered as an extension of a real number and as a real subset of the real line $\Re$ [7]. As a coefficient an interval also signifies the extent of tolerance (or a region) that the parameter can possibly take. However, in decision problems its use is not much attended as its merits.

Let's refer [11] here a very good example of using interval numbers in an optimization problem:

> There are 1000 chickens raised in a chicken farm and they are raised with two kinds of forages – soya and millet. It is known that each chicken eats 1.000–1.130 kg of forage every day and that for good weight gain it needs at least 0.21–0.23 kg of protein and 0.004–0.006 kg of calcium everyday. Per kg of soya contains 48–52% protein and 0.3–0.8% calcium and its price is 0.38–0.42 Yuan. Per kg of millet contains 8.5–11.5% protein and 0.3% calcium and its price is 0.20 Yuan.
> How should the forage be mixed in order to minimize expense on forage?

Most of the parameters used in this problem are inexact and perhaps appropriately given in terms of simple intervals. In reality inexactness of this kind can be cited in countless numbers.

The optimization problem can be structured as follows:

$$
\begin{aligned}
\text{Minimize}\quad & Z=[0.38,0.42]x_1+0.20x_2\\
\text{subject to}\quad & x_1+x_2=[1,1.130]\times 1000,\\
& [0.48,0.52]x_1+[0.085,0.115]x_2\\
& \quad \geqslant [0.21,0.23]\times 1000,\\
& [0.005,0.008]x_1+0.003x_2\\
& \quad \geqslant [0.004,0.006]\times 1000,\\
& x_1,x_2\geqslant 0.
\end{aligned}
$$

However, for solution, techniques of classical linear programming cannot be applied if and unless the above interval-valued structure of the problem be reduced into a standard linear programming structure and for that we have to clear up the following main issues:

- First, regarding interpretation and realization of the inequality relations involving interval coefficients.
- Second, regarding interpretation and realization of the objective 'Min' with respect to an inexact environment.

In this paper, we concentrate on a satisfactory solution approach based on DM's interpretation of inequality relations and objective of the problem with respect to the inexact environment.

This paper is organized as follows. In Section 2 notations of interval number and the interval arithmetics are briefly explained. Section 3 along with its four subsections, give an elaborate study on inequality relation with interval coefficient in search of interpreting and realizing the relation as a constraint of an optimization problem defined in an inexact environment. Section 4 describes the solution principle of an interval linear programming problem, and solution to a previously cited problem [11] and efficiency of our methodology. Section 5 includes the concluding remarks and the future scope.

2. The basic interval arithmetic

All lower case letters denote real numbers and the upper case letters denote the interval numbers or the closed intervals on $\Re$.

2.1. $A=[a_L,a_R]=\{a\colon a_L\leqslant a\leqslant a_R, a\in\Re\}$, where a_L and a_R are left and right limit of the interval A on the real line $\Re$, respectively. If $a_L=a_R$, then $A=[a,a]$ is a real number.

Interval A is alternatively represented as $A=\langle m(A),w(A)\rangle$ where, $m(A)$ and $w(A)$ are the mid-point and half-width (or simply be termed as 'width') of interval A, i.e.,

$$m(A)=\tfrac{1}{2}(a_L+a_R),\qquad w(A)=\tfrac{1}{2}(a_R-a_L).$$

2.2. Let $*\in\{+,-,.,\div\}$ be a binary operation on the set of real numbers.

Then, $A\circledast B=\{a*b,\ a\in A,\ b\in B\}$ defines a binary operation on the set of closed intervals. In case of division it is assumed that $0\notin B$.

If λ is a scalar, then

$$\lambda.A=\lambda[a_L,a_R]=\begin{cases}\lambda[a_L,a_R] & \text{for } \lambda\geqslant 0,\\ \lambda[a_R,a_L] & \text{for } \lambda<0.\end{cases}$$