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Fuzzy Sets and Systems 119 (2001) 41–48

FUZZY sets and systems

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Formulation of general possibilistic linear programming problems for complex industrial systems

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Received December 1997; received in revised form October 1998

Abstract

The current models and methods for PLP are usually restricted on some special types and usually the same type of possibilistic distribution. This paper focuses on linear programming problems with general possibilistic resources (GRPLP) and linear programming problems with general possibilistic objective coefficients (GOPLP). By introducing some new concepts of the largest most possible point, the smallest most possible point, the most optimistic point and the most possible decision, a new approach for formulating possibilistic constraints through general possibilistic resources and a satisfactory solution method will be discussed in this paper. A most possible decision method for GRPLP and a dual approach for GOPLP will be proposed for solving complex industrial decision problems. © 2001 Elsevier Science B.V. All rights reserved.

Keywords: Mathematical programming; Decision making; Possibilistic theory; The largest most possible point

1. Introduction

Since Zadel [16] proposed the theory of possibility, there has been active research on possibilistic linear programming (PLP) problems [2,3,5–10]. As a special term of Fuzzy linear programming (FLP), PLP deals with the linear progamming problems with imprecise coefficients restricated by possibilistic distribution. Many approaches have been proposed to FLP, including symmetrical and asymmetric approach [4,6,17,18]. In general, FLP is associated with fuzzy data which is usually formulated by subjective preference-based membership function, while PLP deals with imprecise data modelled by possibilistic distribution on subjective or objective bases. From this point of view, PLP is usually distinguished and classified from FLP by many researchers [6] and some approaches for FLP may not be applicable for PLP. On the other hand, unlike stochastic linear programming, PLP not only provides computational efficiency and flexible doctrine [6], but also supports possibilistic decision making under environments of uncertainty.

In general, PLP is classified into five types of problems as follows [6]:

(1) LP problems with imprecise resource $\tilde{b}$ (PLP-1).
(2) LP problems with imprecise objective coefficients $\tilde{c}$ (PLP-2).

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PII: S0165-0114(98)00466-7

(3) LP problems with imprecise resource $\tilde{b}$ and technical coefficients $\tilde{A}$ (PLP-3).
(4) LP problems with imprecise resource $\tilde{b}$/technical coefficients $\tilde{A}$ and objective coefficients $\tilde{c}$ (PLP-4).
(5) LP problems with imprecise resource $\tilde{b}$, objective coefficients $\tilde{c}$ and technical coefficients $\tilde{A}$ (PLP-5).

According to Bellmann and Zadeh [1], the objective and constraints play the same roles in fuzzy mathematical programming, i.e. they are in the symmetrical position, which indicates that the dual concept of fuzzy constraints is fuzzy objective and reciprocal [13]. Verdegay [13] discussed the dual relationship between FLP with fuzzy objective and FLP with fuzzy constraints. From this point of view, as a special case of FLP with fuzzy resource, PLP with imprecise resource (PLP-1) is the dual problem of PLP-2, and vice-versa. The solution to PLP-1 may usually be transferred into the solution to PLP-2. Lai and Hwang [7] developed an approach which transformed the PLP-2 into a crisp multiple objective linear programming (MOLP) by introducing the concepts of most possible value, most pessimistic value and most optimistic value, under the assumption that $\tilde{c}$ is a triangular number. Rommelfanger et al. [9] considered $\tilde{c}$ as an interval number and transformed PLP-2 into a LP with number of $2r$ constraints by selecting the number of r possibilistic level preferred by the decision makers (DM). Ramik and Rimanek et al. [8] assumed that $\tilde{b}$ and $\tilde{A}$ are all trapezoidal numbers, and developed a method to solve PLP-3. Tanaka [10] discussed PLP-3 under the consideration of triangular numbers. Dubois [5] proposed the concepts of hard constraint and soft constraint for PLP-3 with interval numbers. Lai and Hwang [7] developed a method for solving PLP-5 based on triangular numbers. Buckley [3] transformed the PLP-5 into a parametric LP by defining possibilistic distribution and conditioned possibilistic distribution for objective function, under the assumption that $\tilde{c}$, $\tilde{b}$ and $\tilde{A}$ are all trapezoidal numbers.

The above methods for solving PLP are largely limited to the range of fuzzy numbers with some special kinds of possibilistic distribution, such as triangular numbers [7,10], interval numbers [5,9], trapezoidal numbers [2,3,8]. Moreover, they are all assumed to be of the same type of normal fuzzy numbers. However, PLP with general or different types of possibilistic distribution and non-normal fuzzy numbers are rarely discussed, and they cannot be solved by the above methods. In many practical problems, especially in complex industrial systems, there exist many kinds of imprecise data, such as resources availability, manufacturing capacity, market demands, unit costs for/benefits from new products or projects, etc. which may not always be readily described as interval numbers, triangular numbers, trapezoidal numbers. Sometimes, it is more suitable to formulate the problems as imprecision with general possibilistic distribution. Therefore, research on PLP with general possibilistic distribution is not only very important in theory, but also valuable to the application of PLP in real-life problems, particularly in complex industrial systems.

This paper focuses on two types of PLP with general possibilistic distribution, i.e. the linear programming problems with general possibilistic resources (GRPLP) and the linear programming problems with general possibilistic objective coefficients (GOPLP). It attempts to propose a new approach to formulating them by introducing the concepts of the largest most possible point, the smallest most possible point and the most possible decision, etc. Two methods for GRPLP and a dual-based approach for GOPLP are developed.

The rest of this paper is organized as follows: Section 2 explains the formulation of GRPLP problems by introducing the new concepts of the largest most possible point, most optimistic point and most pessimistic point, which are different from the ones defined by Lai and Hwang [6]. Based on the above formulation, the satisfactory solution method and the most possible decision method are developed for GRPLP. Then in Section 3, a dual-based approach for GOPLP problems is proposed, and finally the conclusion is given in the Section 4.

2. Linear programming problems with general possibilistic resource (GRPLP)

In real-life industrial problems, the available quantity of resource over a period is usually imprecise when described by interval numbers, trapezoidal numbers or triangular numbers. Sometimes they are expressed as a value with general possibilistic distribution and