PERGAMON

Mathematical and Computer Modelling 34 (2001) 411–422

MATHEMATICAL AND COMPUTER MODELLING

www.elsevier.nl/locate/mcm

An Interior-Point Approach for Solving MC^2 Linear Programming Problems

YIHUA ZHONG[†]
Department of Computer Science
Southwest Petroleum Institute 637001
Nanchong, Sichuan, P.R. China

YONG SHI[‡]
College of Information Science and Technology
6001 Dodge Street, Omaha, NE 68182-0116, U.S.A.
yshi@mail.unomaha.edu

(Received and accepted May 2000)

Abstract—This paper presents an interior-point method to solve the multiple criteria and multiple constraint level linear programming (MC^{2}LP) problems. This approach utilizes the known interior-point method to multiple criteria linear programming (MCLP) and a convex combination method to generate potential solutions for the MC^{2}LP problems. This method can be used as an alternative to the well-known MC^2-simplex method. The numerical comparison study of two methods is provided in the paper.

Keywords—Interior-point method, MC^2 linear programming, Efficient solution, Potential solution.

1. INTRODUCTION

Multiple criteria and multiple constraint level linear programming (MC^{2}LP) is an extension of linear programming (LP) and multiple criteria linear programming (MCLP). Many academicians and practitioners have been devoting to research and solve the problems of LP, MCLP, and MC^{2}LP. There have been numerous research results in theory and practice. The well-known results among these achievements are simplex method, MC-simplex method, and MC^2-simplex method. The simplex method moves toward a solution on the exterior of the constraint polytope, following its vertices. This method has been an extremely efficient computational tool solving linear programming problems ever since it was introduced by Dantzig in 1947. However, the worst case analysis shows that the simplex method and its variants may take an exponential number (depending on the problem size) of pivots to reach an optimal solution, and the method may become impractical in solving very large size general linear programming problems. Therefore, research work has been directed to find an algorithm for linear programming with polynomial complexity. With the introduction of a new algorithm proposed by Karmarkar in 1984 [1],

[†]Visiting professor at University of Nebraska at Omaha.
[‡]Author to whom all correspondence should be addressed.

0895-7177/01/$ - see front matter © 2001 Elsevier Science Ltd. All rights reserved. Typeset by $\mathcal{A}\mathcal{M}\mathcal{S}$-TEX
PII: S0895-7177(01)00072-3

a new computational tool became available for linear programming problems. In contrast to the simplex algorithm, this algorithm moves through the interior of the polytope. This new algorithm and subsequent additional variants established a new class of algorithms for linear programming (see, for example, [2,3]). Since this occured, it has been rivaling the simplex method in theory and practice. The MC-simplex method [4–6] is based on the well-established simplex algorithm for guiding the exploration of multiple criteria and their inherent trade-offs. As the problem size increases, the MC-simplex method based on the simplex algorithm and its vertex information may prolong the search for an acceptable multiobjective solution due to the large number of vertices. Furthermore, methods based on vertex information may encounter difficulties in identifying solutions that are located on a face of the polytope rather than its vertex. Interior-point methods proceed through the interior of the polytope rather than along its vertices, and this may prove less sensitive to problem size and the number of vertices. Therefore, interior-point approaches to MCLP problems are also studied [7–10]. $\mathrm{MC^2LP}$ problems have been completely discussed [11–14]. Seiford and Yu [12] proposed an $\mathrm{MC^2}$-simplex method for solving $\mathrm{MC^2LP}$ problems. However, when applying this approach to solve real-world problems with $\mathrm{MC^2}$ factors, one needs a large amount of time to find all potential solutions. For example, during decision making, the decision maker may hesitate to choose a particular potential solution from a set of potential solutions. The $\mathrm{MC^2}$-simplex method may also confront similar difficulties as the simplex method and the MC-simplex method.

In this paper, we present an alternative approach for finding a potential solution of an $\mathrm{MC^2LP}$ problem using interior-point approaches to MCLP and a convex combination. This method is different from the usual solution technique of the $\mathrm{MC^2}$-simplex method. After splitting an $\mathrm{MC^2LP}$ problem into p MCLP problems by partitioning the columns of the right-hand side of the matrix of the $\mathrm{MC^2LP}$ problem, we solve these p MCLP problems through the interior-point method to MCLP problems. Then, we combine their solutions by convex combination to produce a potential solution of the $\mathrm{MC^2LP}$ problem.

This paper proceeds as follows. In Section 2, we briefly review $\mathrm{MC^2LP}$ problems. In Section 3, we sketch interior-point approaches to MCLP problems. In Section 4, we present an interior-point method for $\mathrm{MC^2LP}$ problems. Then, a numerical example is used to illustrate this approach. In Section 5, we conclude this paper with some further research problems.

2. $\mathrm{MC^2LP}$ PROBLEMS

As the development of mathematical programming theory and decision science advance, linear programming (LP) with a single criteria (objective) and a single (fixed) resource availability level (right-hand side) was extended as multiple criteria linear programming (MCLP) with multiple conflicting criteria. MCLP was further extended as multiple criteria and multiple constraint level linear programming ($\mathrm{MC^2LP}$). In the framework of $\mathrm{MC^2LP}$ problems, multiple (discrete) levels of resource availability are explicitly expressed. This model is rooted in both theoretical and practical facts. First, from the linear system structure's point of view, the criteria and constraints may be "interchangeable". Thus, like multiple criteria, multiple constraint (resource availability) levels can be considered. Second, from the application's point of view, it is more realistic to consider multiple resource availability levels (discrete right-hand sides) than a single resource availability level in isolation. The philosophy behind this perspective is that the availability of the resources can fluctuate depending on the decision situation forces, such as the desirability levels believed by the different decision committee members. The relationship between $\mathrm{MC^2LP}$ and MCLP can be found in [15]. To facilitate our discussion, we will simply review the model of $\mathrm{MC^2LP}$ problems that are studied in this paper as follows.

An $\mathrm{MC^2LP}$ problem can be formulated as

$$\max \lambda^{\mathsf{T}} \mathbf{C}\mathbf{x}, \quad \text{s.t. } \mathbf{A}\mathbf{x} = \mathbf{D}\gamma, \qquad \mathbf{x} \geq \mathbf{0}, \tag{1}$$