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Measuring the satisfaction of constraints in fuzzy linear programming

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Abstract

The paper proposes a new kind of method for solving fuzzy linear programming problems based on the satisfaction (or fulfillment) degree of the constraints. Using a new ranking method of fuzzy numbers, the fulfillment of the constraints can be measured. Then the properties of the ranking index are discussed. With this ranking index, the decision maker can make the constraints tight or loose based on his optimistic or pessimistic attitude and get the optimal solution from the fuzzy constraint space. The corresponding value of objective distribution function can be obtained. A numerical example illustrates the merits of the approach. © 2001 Published by Elsevier Science B.V.

Keywords: Fuzzy numbers; Fuzzy mathematical programming; Fuzzy constraint satisfaction

1. Introduction

The research on fuzzy mathematical programming has been an active area since Bellman and Zadeh proposed the definition of fuzzy decision making [3,10,17,18,21]. A recent review was given by Rommelfanger [22]. The most commonly used method is to transform it into one or a series of crisp programming problems. Recently, Buckley proposed the joint solution method for fuzzy linear programming problem [4]. Julien transformed the fuzzy linear programming problem with the best and the worst linear programming problem at different α cut levels, and got the possibility distribution of the optimal objective value [14]. Tong also used a similar idea and got a preferred optimal solution when the coefficients are interval numbers [25], but none of them discuss the relationship between the optimal solution and the fuzzy constraints.

In this paper, using the concept of fuzzy numbers comparison, we propose a new method for solving fuzzy number linear programming problems. The relationship between the optimal solution and the fulfillment of fuzzy constraint is established, and the corresponding values of the optimal objective distribution function

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can be obtained. With the constraints fully, partially or not satisfied, different crisp solutions can be obtained according to the decision maker's optimistic or pessimistic attitude.

For the comparison of fuzzy numbers, there are many different methods [7,9,11,12,15,18,20,23,24,26,27]. The best-known indexes come from the possibility theory of Dubois and Prade [8,9], based on possibility measure and necessity measure; they proposed four indexes of inequality between fuzzy numbers. Ichihashi and Tanaka proposed a kind of more detailed index for interval numbers than that of Dubois and Prade [13]. Ramik and Remelfanger proposed multivalued index for left and right trapezoidal fuzzy numbers to solve the fuzzy linear programming problems [21]. Liou and Fortemps proposed a fuzzy number ranking approach with their total integral values or area compensation [11,12,15]. Tsueng proposed a ranking index based on dominance and indifference areas of fuzzy numbers, and applied to evaluate the alternatives in decision-making problems [26].

All these methods can be classified into two kinds: One is using crisp relations to rank fuzzy numbers [7,11,12,15,20,23], every fuzzy number is mapped to a point on the real line; the other is to use fuzzy relations to rank fuzzy numbers [9,18,24,26,27], the relations are interpreted as fuzzy membership functions.

Obviously, to measure the fulfillment of the constraints in fuzzy linear programming, the latter is more reasonable. Recently, Baoding and Kakuzo proposed fuzzy chance programming problem with the possibility index to measure the fulfillment of the fuzzy constraints [2]. Nakahara also proposed a more generalized ranking method than the possibility index with two ranking criteria [18]. With fuzzy number ranking indexes, the treatment of fuzzy constraints can be proposed. But all these indexes are only direct use or the extension of the possibility index, they cannot distinguish fuzzy numbers in some conditions. Furthermore, because they are based on different criteria, they are inconsistent in the comparison of results. In this paper, the ranking method of Tseng [26] is extended. A new kind of inequality index between fuzzy numbers is proposed, its properties are discussed, and it is applied to measure the fulfillment of the constraints in fuzzy linear programming problem. Besides, the optimal solutions with different constraint satisfaction degrees can be obtained. The interval number comparison index proposed by Nakahara becomes a special case of the proposed method.

2. Preliminaries

First, the concepts of fuzzy numbers are introduced. A fuzzy number is a fuzzy subset of the real line, which is both "normal" and "convex". For a fuzzy number $\tilde{a}$, its membership can be denoted by

$$\mu_{\tilde{a}}(x) = \begin{cases} L(x), & x < m, \\ 1, & m \leqslant x \leqslant n, \\ R(x), & x > n, \end{cases} \tag{1}$$

where $L(x)$ is a upper semi-continuous, strictly increasing function for $x < m$ and there exists $m_1 < m$ such that $L(x) = 0$ for $x \leqslant m_1$, $R(x)$ is continuous, strictly decreasing function for $x > n$ and there exists $n_1 > n$ such that $R(x) = 0$ for $x \geqslant n_1$, and $L(x), R(x)$ are called the left reference function and right reference function, respectively. And the interval number of α-cut $\tilde{a}(\alpha) = \{x \in R \mid \mu_{\tilde{a}}(x) \geqslant \alpha\}$ can be denoted by $[a^{\mathrm{L}}_{\alpha}, a^{\mathrm{R}}_{\alpha}]$.

In practical fuzzy mathematical programming problem, interval numbers, triangular fuzzy numbers and trapezoidal fuzzy numbers are most commonly used, because they have intuitive appeal and can be easily specified by the decision maker. So we will mainly discuss the ranking method of interval numbers, symmetrical triangular fuzzy numbers and trapezoidal fuzzy numbers.